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NONLINEAR SYSTEM OF IMPULSIVE INTEGRO-DIFFERENTIAL EQUATIONS WITH HILFER FRACTIONAL OPERATOR AND MIXED MAXIMA

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A nonlocal boundary value problem for a system of ordinary integro-differential equations with impulsive effects, nonlinear mixed maxima and fractional order Hilfer operator is investigated. The nonlinear boundary value condition is given in the nonlinear integral form. The problem is reduced to the nonlinear system of functional integral equations. The system of functional integral equations has terms of nonlinear functions in integral and non-integral forms. The method of successive approximations in a combination with the method of compressing mapping is used in proving the unique solvability of the boundary value problem.

Keywords: *impulsive integro-differential equation, Hilfer fractional operator, nonlocal boundary condition, mixed maxima, unique solvability.*

1. Introduction

Fractional calculus plays an important role in the mathematical modeling of many problems in scientific and engineering disciplines, continuum and statistical mechanics, in the construction of the Ebola epidemic model, a fractional model for the dynamics of the tuberculosis infection and novel coronavirus (nCoV-2019) and otheres [1–9]. In the applications of fractional derivatives in solving differential equations were obtained interesting results in the many papers (see, for exmample, [10–19]. Many problems in applications are described by fractional order differential equations, the solution of which is a function with first kind discontinuities at some points of the time interval. Fractional differential equations of such type are called fractional differential equations with impulsive effects. Integer order differential equations with impulsive effects are considered, in particular, in [20–27]. On the other hand, in recent years the interest in the studying of differential equations with nonlocal boundary value conditions is increasing (see, for example, [28–37]). Also a lot of publications in studying of differential equations with impulsive effects, describing many natural and practical processes, are appearing [38–45]. In [46–49], the issues of the solution existence for impulsive differential equations with fractional order derivatives are considered.

Differential equations with maxima have singularities in single valued solvability. Moreover, the jumpiness of solutions is a natural thing for differential equations with mixed maxima [50]. In the present paper, we investigate a nonlinear nonlocal boundary value problem for a system of ordinary Hilfer type fractional order nonlinear integro-differential equations with impulsive effects and nonlinear mixed maxima. The questions of existence and uniqueness of the solution of the nonlinear boundary value problem are investigated. This paper is the further development of the paper [51].

Let $(0,T)$ be an interval, $0<T<\infty$, $\alpha>0$. The Riemann — Liouville α-order fractional integral of a function $\eta(t)$ is defined as follows:

$$I_{0t}^{\alpha}\eta(t)=\frac{1}{\Gamma(\alpha)}\int\limits_0^t (t-s)^{\alpha-1}\eta(s)ds,\quad \alpha>0,\quad t\in(0,T),$$

where $\Gamma(\alpha)$ is the Gamma-function. Let $n-1<\alpha\leqslant n\in\mathbb{N}$. The Riemann — Liouville α-order fractional derivative of a function $\eta(t)$ is defined as

$$D_{0t}^{\alpha}\eta(t)=\frac{d^n}{dt^n}I_{0t}^{n-\alpha}\eta(t),\quad t\in(0,T).$$

The Hilfer operator will be considered in the form

$$D_{0t}^{\alpha,\gamma}=I_{0t}^{\gamma-\alpha}\frac{d}{dt}I_{0t}^{1-\gamma},\quad 0<\alpha\leqslant\gamma\leqslant 1.$$

Note that this operator can be express through operator $\mathcal{D}_{0t}^{\alpha,\beta}$ from [4, vol. 1, p. 55, formula (37)] by the equality $D_{0t}^{\alpha,\gamma}=\mathcal{D}_{0t}^{\alpha,\frac{\gamma-\alpha}{1-\alpha}}$, i. e. $\gamma=\alpha+\beta(1-\alpha)$.

2. Problem statement

On the set $(0,T)\setminus\{t_i\}$, $i=1,2,\ldots,p$ for $0<\alpha\leq\gamma\leqslant 1$ we consider the following fractional order system of nonlinear integro-differential equations

$$D_{0t}^{\alpha,\gamma}x(t)=f\left(t,x(t),\,I_{0T}^{1}\Theta\left(t,s,\max\left\{x(\tau)|\tau\in\left[h_1:|:h_2\right]\right\}\right)\right),\tag{1}$$

where $\left[h_1:|:h_2\right]=\Big\{\left[\min\left\{h_1(t);h_2(t)\right\};\max\left\{h_1(t);h_2(t)\right\}\right]:t\in(0,T)\setminus\{t_i\},\ i=1,2,\ldots,p,\ h_j(t)=h_j(t,x(t)),\ j=1,2\Big\}$, $f:[0,T]\times\mathbb{R}^n\times\mathbb{R}^n\to\mathbb{R}^n$, $\Theta:[0,T]^2\times\mathbb{R}^n\to\mathbb{R}^n$.

We study the unique solvability of the system of nonlinear integro-differential equations (1) with nonlinear nonlocal boundary value condition

$$A\cdot I_{0t}^{1-\gamma}x(0^+)+\Gamma(\gamma)I_{0T}^{1}\left[K(t,s)\,s^{1-\gamma}x(s)\right]=B(t,x(t))\tag{2}$$

and nonlinear impulsive effect condition

$$\frac{t_{i+1}^{\gamma-1}}{\Gamma(\gamma)}I_{t_i\,t_{i+1}}^{1-\gamma}x\left(t_i^+\right)-x\left(t_i^-\right)=F_i\big(x\left(t_i\right)\big),\qquad i=1,2,\ldots,p,\tag{3}$$

where $0=t_0<t_1<\cdots<t_p<t_{p+1}=T$, $A\in\mathbb{R}^{n\times n}$ is given matrix, $K(t,s)$ is given $(n\times n)$-dimensional matrix function and $\det Q(t)\neq 0$, $Q(t)=A+\int\limits_0^T K(t,s)ds$, $F_i:\mathbb{R}^n\to\mathbb{R}^n$, $B:[0,T]\times\mathbb{R}^n\to\mathbb{R}^n$ are given functions; $I_{0t}^{1-\gamma}x(0^+)=\lim\limits_{t\to 0^+}I_{0t}^{1-\gamma}x(t)$, $x\left(t_i^+\right)=\lim\limits_{l\to 0^+}x\left(t_i+l\right)$, $x\left(t_i^-\right)=\lim\limits_{l\to 0^+}x\left(t_i-l\right)$ are right-hand sided and left-hand sided limits of function $x(t)$ at the point $t=t_i$, respectively.

2.1. Formulation of problem.

To find the function $x(t) \in \mathbb{R}^n$, such that for all $t \in (0,T) \setminus \{t_i\},\ i = 1, 2, \dots, p,\ 0 < t_1 < t_2 < \dots < t_p < T$ it satisfies integro-differential equation (1), nonlinear integral condition (2) and for $t = t_i,\ i = 1, 2, \dots, p$ nonlinear limit conditions (3) hold for it.

3. Reduction the problem to a functional integral equation

Let a function $x(t) \in \mathbb{R}^n$ is a solution of nonlocal boundary value problem (1)–(3) on $(0,T) \setminus \{t_i\}$, $i = 1, 2, \dots, p$. We rewrite fractional differential equation (1) on the interval $(0, t_1]$ as

$$I_{0\,t_1}^{\gamma-\alpha} \frac{d}{dt} I_{0\,t_1}^{1-\gamma} x(t) = f\left(t, x, y\right),$$

where by $f(t, x, y)$ we denote the given nonlinear function

$$f\left(t, x(t),\, I_{0\,T}^{1} \Theta\left(t, s, \max\left\{x(\tau) | \tau \in \left[h_1(t, x(t)) : | : h_2(t, x(t))\right]\right\}\right)\right).$$

Appling the operator $I_{0\,t_1}^{\alpha}$ to the both sides of the last differential equation, we obtain

$$I_{0\,t_1}^{\gamma} \frac{d}{dt} I_{0\,t_1}^{1-\gamma} x(t) = \frac{1}{\Gamma(\alpha)} \int\limits_0^{t_1} (t_1 - s)^{\alpha-1} f\left(s, x, y\right)\, ds.$$

Hence, taking into account the formula

$$I_{0\,t_1}^{\gamma} \frac{d}{dt} I_{0\,t_1}^{1-\gamma} x(t) = x(t_1^-) - \frac{t_1^{\gamma-1}}{\Gamma(\gamma)} \lim_{t \to 0^+} I_{0\,t_1}^{1-\gamma} x(t),$$

we obtain

$$x\left(t_1^-\right) = \frac{t_1^{\gamma-1}}{\Gamma(\gamma)} \lim_{t \to 0^+} I_{0\,t_1}^{1-\gamma} x(t) + \frac{1}{\Gamma(\alpha)} \int\limits_0^{t_1} (t_1 - s)^{\alpha-1} f\left(s, x, y\right)\, ds, \quad t \in (0,\, t_1].$$

Analogously, by integration of the fractional order differential equation (1) on the intervals $(t_1, t_2]$, $(t_2, t_3]$, $\dots$, $(t_p, t_{p+1}]$, we obtain:

$$x\left(t_2^-\right) = \frac{t_2^{\gamma-1}}{\Gamma(\gamma)} \lim_{t \to t_1^+} I_{t_1\,t_2}^{1-\gamma} x(t) + \frac{1}{\Gamma(\alpha)} \int\limits_{t_1}^{t_2} (t_2 - s)^{\alpha-1} f\left(s, x, y\right)\, ds, \quad t \in (t_1, t_2],$$

$$x(t_3^-) = \frac{t_3^{\gamma-1}}{\Gamma(\gamma)} \lim_{t \to t_2^+} I_{t_2\,t_3}^{1-\gamma} x(t) + \frac{1}{\Gamma(\alpha)} \int\limits_{t_2}^{t_3} (t_3 - s)^{\alpha-1} f\left(s, x, y\right)\, ds, \quad t \in (t_2, t_3],$$

$$\dots$$

$$x\left(t_{p+1}^-\right) = \frac{t_{p+1}^{\gamma-1}}{\Gamma(\gamma)} \lim_{t \to t_p^+} I_{t_p\,t_{p+1}}^{1-\gamma} x(t) + \frac{1}{\Gamma(\alpha)} \int\limits_{t_p}^{t_{p+1}} (t_{p+1} - s)^{\alpha-1} f\left(s, x, y\right)\, ds, \quad t \in (t_p, t_{p+1}].$$

Hence, taking into account the equality $x(t_{k+1}^-) = x(t)$ on the interval $(0, T]$, we have

$$\frac{1}{\Gamma(\alpha)} \int\limits_0^{t} (t - s)^{\alpha-1} f(s, x, y)\, ds = \left[x\left(t_1\right) - \frac{t_1^{\gamma-1}}{\Gamma(\gamma)} \lim_{t \to 0^+} I_{0\,t_1}^{1-\gamma} x(t)\right] +$$

$$+\left[x\left(t_2\right)-\frac{t_2^{\gamma-1}}{\Gamma\left(\gamma\right)}\lim_{t\to t_1^+}I_{t_1\,t_2}^{1-\gamma}\,x(t)\right]+\ \ldots\ +\left[x(t)-\frac{t_{p+1}^{\gamma-1}}{\Gamma\left(\gamma\right)}\lim_{t\to t_p^+}I_{t_p\,t_{p+1}}^{1-\gamma}\,x(t)\right]=$$

$$=-\frac{t_1^{\gamma-1}}{\Gamma\left(\gamma\right)}\lim_{t\to 0^+}I_{0\,t_1}^{1-\gamma}\,x(t)-\left[\frac{t_2^{\gamma-1}}{\Gamma\left(\gamma\right)}\lim_{t\to t_1^+}I_{t_1\,t_2}^{1-\gamma}\,x(t)-x\left(t_1\right)\right]-\left[\frac{t_3^{\gamma-1}}{\Gamma\left(\gamma\right)}\lim_{t\to t_2^+}I_{t_2\,t_3}^{1-\gamma}\,x(t)-x\left(t_2\right)\right]-$$

$$-\ldots-\left[\frac{t_{p+1}^{\gamma-1}}{\Gamma\left(\gamma\right)}\lim_{t\to t_p^+}I_{t_p\,t_{p+1}}^{1-\gamma}x(t)-x\left(t_p\right)\right]+x(t).$$

Due to condition (3), the last equality we rewrite as

$$x(t)=\frac{t^{\,\gamma-1}}{\Gamma\left(\gamma\right)}\lim_{t\to 0^+}I_{0\,t}^{1-\gamma}\,x(t)+\frac{1}{\Gamma\left(\alpha\right)}\int_0^t(t-s)^{\alpha-1}f(s,x,y)\,ds+\sum_{0<t_i<t}F_i\left(x\left(t_i\right)\right). \qquad (4)$$

The last equality has singularity at the point $t=0$. We subordinate the function $x(t)\in\mathbb{R}^n$ in (4) to satisfy boundary value condition (2). Then, multiplying both sides of obtained equality by $t^{\,1-\gamma}$, presentation (4) we rewrite as

$$\Gamma\left(\gamma\right)t^{\,1-\gamma}x(t)=I_{0\,t}^{1-\gamma}\,x(0^+)+\frac{\Gamma\left(\gamma\right)t^{\,1-\gamma}}{\Gamma\left(\alpha\right)}\int_0^t(t-s)^{\alpha-1}f(s,x,y)\,ds+$$

$$+\Gamma\left(\gamma\right)t^{\,1-\gamma}\sum_{0<t_i<t}F_i\left(x\left(t_i\right)\right).$$

Now we multiple the last equality by $K(t,s)$ and integrate it from 0 to T. Then

$$\Gamma\left(\gamma\right)\int_0^T K(t,s)\,s^{\,1-\gamma}x(s)\,ds=I_{0\,t}^{1-\gamma}\,x(0^+)\int_0^T K(t,s)\,ds+$$

$$+\frac{\Gamma\left(\gamma\right)}{\Gamma\left(\alpha\right)}\int_0^T K(t,s)\,s^{\,1-\gamma}\int_0^s(s-\theta)^{\alpha-1}f(\theta,x,y)\,d\theta\,ds+$$

$$+\,\Gamma\left(\gamma\right)\int_0^T K(t,s)\,s^{\,1-\gamma}\sum_{0<t_i<t}F_i\left(x\left(t_i\right)\right)\,ds. \qquad (5)$$

Substituting presentation (5) into nonlinear condition (2), we obtain:

$$\left[A+\int_0^T K(t,s)\,ds\right]I_{0\,t}^{1-\gamma}x(0^+)=$$

$$=B(t,x(t))-\frac{\Gamma\left(\gamma\right)}{\Gamma\left(\alpha\right)}\int_0^T K(t,s)\,s^{\,1-\gamma}\int_0^s(s-\theta)^{\alpha-1}f(\theta,x,y)\,d\theta\,ds-$$

$$-\,\Gamma\left(\gamma\right)\int_0^T K(t,s)\,s^{\,1-\gamma}\sum_{0<t_i<t}F_i\left(x\left(t_i\right)\right)\,ds. \qquad (6)$$

By virtue of $\det Q(t) = \det \left[A + \int\limits_0^T K(t,s)ds \right] \neq 0$, the equality (6) one can rewrite as

$$I_{0\,t}^{1-\gamma} x(0^+) = Q^{-1}(t)B(t,x(t)) - \\ - \frac{\Gamma(\gamma)}{\Gamma(\alpha)} Q^{-1}(t) \int\limits_0^T K(t,s)\, s^{1-\gamma} \int\limits_0^s (s-\theta)^{\alpha-1} f(\theta,x,y)\, d\theta\, ds - \\ - \Gamma(\gamma) Q^{-1}(t) \int\limits_0^T K(t,s)\, s^{1-\gamma} \sum_{0<t_i<t} F_i\left(x\left(t_i\right)\right)\, ds. \quad (7)$$

Substituting equality (7) into representation (4), we obtain

$$x(t) = \frac{1}{\Gamma(\gamma)} t^{\gamma-1} Q^{-1}(t) B(t,x(t)) + \frac{1}{\Gamma(\alpha)} \int\limits_0^t (t-s)^{\alpha-1} f(s,\, x,y) ds - \\ - \frac{1}{\Gamma(\alpha)} t^{\gamma-1} Q^{-1}(t) \int\limits_0^T K(t,s)\, s^{1-\gamma} \int\limits_0^s (s-\theta)^{\alpha-1} f(\theta,x,y)\, d\theta\, ds - \\ - t^{\gamma-1} Q^{-1}(t) \int\limits_0^T K(t,s)\, s^{1-\gamma} \sum_{0<t_i<t} F_i\left(x\left(t_i\right)\right)\, ds + \sum_{0<t_i<t} F_i\left(x\left(t_i\right)\right). \quad (8)$$

Since the following equalities hold

$$\int\limits_0^T K(t,s)\, s^{1-\gamma} \int\limits_0^s (s-\theta)^{\alpha-1} f(\theta,\, x,y)\, d\theta ds = \\ = \int\limits_0^T \int\limits_s^T K(t,\theta)\, \theta^{1-\gamma}\, d\theta\, (t-s)^{\alpha-1} f(s,\, x,y)\, ds,$$

$$\int\limits_0^T K(t,s)\, s^{1-\gamma} \sum_{0<t_i<t} F_i\left(x\left(t_i\right)\right)\, ds = \sum_{0<t_i<T} \int\limits_{t_i}^T K(t,s)\, s^{1-\gamma}\, ds\, F_i\left(x\left(t_i\right)\right),$$

presentation (8) implies

$$x(t) = \frac{1}{\Gamma(\gamma)} t^{\gamma-1} Q^{-1}(t) B(t,x(t)) + \frac{1}{\Gamma(\alpha)} \int\limits_0^t (t-s)^{\alpha-1} f(s,\, x,y) ds - \\ - \frac{1}{\Gamma(\alpha)} t^{\gamma-1} Q^{-1}(t) \int\limits_0^T \int\limits_s^T K(t,\theta)\, \theta^{1-\gamma} d\theta\, (t-s)^{\alpha-1} f(s,\, x,y)\, ds - \\ - t^{\gamma-1} Q^{-1}(t) \sum_{0<t_i<T} \int\limits_{t_i}^T K(t,s)\, s^{1-\gamma}\, ds\, F_i\left(x\left(t_i\right)\right) + \sum_{0<t_i<t} F_i\left(x\left(t_i\right)\right). \quad (9)$$

After some simplifications in (9) we obtain that the following equalities hold:

$$\frac{1}{\Gamma(\alpha)}\int\limits_0^t (t-s)^{\alpha-1} f(s,x,y)\,ds-$$

$$-\frac{1}{\Gamma(\alpha)} t^{\gamma-1} Q^{-1}(t)\int\limits_0^T\int\limits_s^T K(t,\theta)\,\theta^{1-\gamma}\,d\theta\,(t-s)^{\alpha-1} f(s,x,y)\,ds=$$

$$=Q^{-1}(t)\frac{1}{\Gamma(\alpha)}\int\limits_0^t\left(A+\int\limits_0^s K(t,\theta)d\theta\right)(t-s)^{\alpha-1} f(s,x,y)\,ds-$$

$$-\frac{1}{\Gamma(\alpha)} t^{\gamma-1} Q^{-1}(t)\int\limits_0^T\int\limits_s^T K(t,\theta)\,\theta^{1-\gamma}\,d\theta\,(t-s)^{\alpha-1} f(s,x,y)\,ds, \quad (10)$$

$$\sum_{0<t_i<t} F_i\left(x\left(t_i\right)\right)-t^{\gamma-1}Q^{-1}(t)\sum_{0<t_i<T}\int\limits_{t_i}^T K(t,s)\,s^{1-\gamma}ds\,F_i\left(x\left(t_i\right)\right)=$$

$$=Q^{-1}(t)\sum_{0<t_i<t}\left(A+\int\limits_0^{t_i} K(t_i,s)ds\right)F_i\left(x\left(t_i\right)\right)-$$

$$-t^{\gamma-1}Q^{-1}(t)\sum_{t<t_{i+1}<T}\int\limits_{t_i}^T K(t,s)\,s^{1-\gamma}ds\,F_i\left(x\left(t_i\right)\right). \quad (11)$$

Taking into account (10), (11) and multiplying both sides by $t^{1-\gamma}$, from presentation (9) it follows the nonlinear functional integral equation

$$t^{1-\gamma}x(t)=J(t;\,x)\equiv\frac{1}{\Gamma(\gamma)}Q^{-1}(t)B(t,x(t))+\sum_{0<t_i<t} t^{1-\gamma}G\left(t,t_i\right)\,F_i\left(x\left(t_i\right)\right)+$$

$$+\frac{1}{\Gamma(\alpha)}\int\limits_0^T t^{1-\gamma}G(t,s)(t-s)^{\alpha-1}f\left(s,x(s),\int\limits_0^T\Theta\left(s,\theta,\max\left\{x(\tau)\middle|\tau\in\left[h_1:|:h_2\right]\right\}\right)d\theta\right)ds \quad (12)$$

for $t\in(t_i,t_{i+1}]$, $i=0,1,\ldots,p$, where $h_j=h_j\,(\theta,\,x(\theta))$, $j=1,2$,

$$G(t,s)=\begin{cases} Q^{-1}(t)\Big(A+\int\limits_0^s K(t,\theta)d\theta\Big), & 0\leqslant s\leqslant t,\\ -t^{\gamma-1}Q^{-1}(t)\int\limits_s^T K(t,\theta)\,\theta^{1-\gamma}d\theta, & t<s\leqslant T.\end{cases}$$

4. One value solvability theorem

According to (12), the unknown function $x(t)$ we consider with the weight function $t^{1-\gamma}$. So, by $C\left([0,T],\mathbb{R}^n\right)$ the Banach space of continuous vector functions $t^{1-\gamma}x(t)$, defined on the segment $[0,T]$, with the norm

$$\left\|\,t^{1-\gamma}x(t)\,\right\|=\sqrt{\sum_{j=1}^n\max_{0\leq t\leq T}|\,t^{1-\gamma}x_j(t)\,|},$$

is denoted. By $PC\left([0,T],\,\mathbb{R}^n\right)$ we denote the following linear vector space

$$PC\left([0,T],\,\mathbb{R}^n\right)=\Big\{t^{1-\gamma}x(t):[0,T]\to\mathbb{R}^n;\ x(t)\in C\big(\left(t_i,\,t_{i+1}\right],\mathbb{R}^n\big),\ i=1,\ldots,p\Big\},$$

where $x\left(t_i^{+}\right)$ and $x\left(t_i^{-}\right)$ $(i=0,1,\ldots,p)$ exist and bounded; $x\left(t_i^{-}\right)=x\left(t_i\right)$. Note, that the linear vector space $PC\left([0,T],\,\mathbb{R}^n\right)$ is Banach space with the norm

$$\left\|\,t^{1-\gamma}x(t)\,\right\|_{PC}=\max\left\{\left\|\,t^{1-\gamma}x(t)\,\right\|_{C((t_i,\,t_{i+1}])},\ i=1,2,\ldots,p\right\}.$$

For the unique solvability of nonlocal boundary value problem (1)–(3) on the set $(0,T)\setminus\left\{t_i\right\},\ i=1,2,\ldots,p$ we study the existence of a unique solution of equation (12) in the class $PC\left([0,T],\,\mathbb{R}^n\right)$.

Theorem 1. *Suppose the following conditions are fulfilled:*

1) $M_f=\max\limits_{0\leqslant t\leqslant T}\left|\,f\left(t,0,\int\limits_0^T\Theta\left(t,s,0\right)\,ds\right)\right|<\infty$;

2) $N_F=\max\limits_{i\in\{1,2,\ldots,p\}}\left|\,F_i(0)\,\right|<\infty;\ \ B_Q=\frac{1}{\Gamma(\gamma)}\max\limits_{0\leqslant t\leqslant T}\left|\,Q^{-1}(t)B\left(t,0\right)\right|<\infty$;

3) *for all* $t\in[0,T],\ x,y\in\mathbb{R}^n$ *and* $i=1,2$ *holds*

$$\left|\,f(t,x_1,y_1)-f(t,x_2,y_2)\,\right|\leqslant M_1(t)\,t^{1-\gamma}\left|\,x_1-x_2\,\right|+M_2(t)\,\left|\,y_1-y_2\,\right|;$$

4) *for all* $(t,s)\in[0,T]^2,\ x\in\mathbb{R}^n$ *holds*

$$\left|\,\Theta(t,s,x_1)-\Theta(t,s,x_2)\,\right|\leqslant M_3(t,s)\,s^{1-\gamma}\left|\,x_1-x_2\,\right|;$$

5) *for all* $t\in[0,T]$, $x\in\mathbb{R}^n$ *holds*

$$\left|\,B(t,x_1)-B(t,x_2)\,\right|\leqslant M_4(t)\,t^{1-\gamma}\left|\,x_1-x_2\,\right|;$$

6) *for all* $t\in[0,T],\ x\in\mathbb{R}^n$ *holds*

$$\left|\,h_j(t,x_1)-h_j(t,x_2)\,\right|\leqslant M_{4+j}(t)\,t^{1-\gamma}\left|\,x_1-x_2\,\right|,\ \ j=1,2;$$

7) *for all* $x\in\mathbb{R}^n,\ \ i=0,1,\ldots,p$ *holds*

$$\left|\,F_i(x_1)-F_i(x_2)\,\right|\leqslant N_i\,t_i^{1-\gamma}\left|\,x_1-x_2\,\right|,\ \ 0<N_i;$$

8) $\rho=S_1+S_2+S_3+S_4<1$, *where* $0<M_i(t)\in C[0,T]$, $i=1,2,\ldots 6$,

$$S_1=\max_{0\leq t\leq T}\int\limits_0^T\left|\,t^{1-\gamma}G(t,s)\,\right|\,(t-s)^{\alpha-1}M_1(s)\,d\,s,$$

$$S_2=\max_{0\leq t\leq T}\int\limits_0^T\left|\,t^{1-\gamma}G(t,s)\,\right|\,(t-s)^{\alpha-1}M_2(s)\int\limits_0^T M_3(s,\theta)\,\big[1+M_f\big(M_5(\theta)+M_6(\theta)\big)\big]\,d\theta ds,$$

$$S_3=\max_{0\leq t\leq T}\sum_{i=1}^{p}\left|t^{1-\gamma}G(t,t_i)\right|\cdot N_i,\quad S_4=\frac{1}{\Gamma\left(\gamma\right)}\max_{0\leq t\leq T}\left|\,Q^{-1}(t)\,\right|M_4(t).$$

Then functional integral equation (12) *has a unique solution in the class* $PC\left([0,\, T], \mathbb{R}^n\right)$. *This solution can be found by the following iterative process:*

$$\begin{cases} t^{1-\gamma} x^m(t) = J(t;\, x^{m-1}), & t \in (t_i,\, t_{i+1}), \quad m = 1, 2, 3, \ldots; \\ t^{1-\gamma} x^0(t) = 0, & t \in (t_i,\, t_{i+1}), \quad i = 0, 1, 2, \ldots, p. \end{cases} \tag{13}$$

Proof. We consider the operator $J : PC\left([0, T];\, \mathbb{R}^n\right) \to PC\left([0, T] \times \mathbb{R}^n\right)$, defined by the right-hand side of equation (12). Taking the first and the second conditions of the theorem, for the first difference of approximations (13) we have the estimates

$$\left\| t^{1-\gamma}\left(x^1(t) - x^0(t)\right) \right\| \leqslant \frac{1}{\Gamma(\gamma)} \max_{0 \leqslant t \leqslant T} \left| Q^{-1}(t) B\left(t, x^0(t)\right) \right| +$$

$$+ \max_{0 \leqslant t \leqslant T} \sum_{i=1}^{p} \left| t^{1-\gamma} G(t, t_i) \right| \left| F_i\left(x^0(t_i)\right) \right| + \max_{0 \leqslant t \leqslant T} \int_0^T \left| t^{1-\gamma} G(t, s) \right| (t-s)^{\alpha-1} \times$$

$$\times \left| f\left(s, x^0(s), \int_0^T \Theta\left(s, \theta, \max\left\{x^0(\tau) | \tau \in \left[h_1^0 : | : h_2^0\right]\right\}\right) d\theta\right) \right| ds \leqslant$$

$$\leqslant S_0\left(M_f + N_F\right) + B_Q < \infty, \tag{14}$$

where $h_j^0 = h_j(\theta, x^0(\theta)) = h_j(\theta, 0), \;\; j = 1, 2,$

$$S_0 = \max_{0 \leq t \leq T} \int_0^T \left| t^{1-\gamma} G(t, s) \right| (t-s)^{\alpha-1} ds + \max_{0 \leqslant t \leqslant T} \sum_{i=1}^{p} \left| t^{1-\gamma} G(t, t_i) \right|,$$

$$M_f = \max_{0 \leqslant t \leqslant T} \left| f\left(t, x^0(t), \int_0^T \Theta\left(t, s, x^0(s)\right) ds\right) \right|,$$

$$B_Q = \frac{1}{\Gamma(\gamma)} \max_{0 \leqslant t \leqslant T} \left| Q^{-1}(t) B\left(t, x^0(t)\right) \right|, \quad N_F = \max_{i \in \{1, 2, \ldots, p\}} \left| F_i\left(x_i^0(t)\right) \right|.$$

Then, by the conditions of the theorem, for difference of arbitrary consecutive approximations and for arbitrary $t \in (t_i, t_{i+1}]$, we have

$$\left| t^{1-\gamma}\left(x^m(t) - x^{m-1}(t)\right) \right| \leqslant \frac{1}{\Gamma(\gamma)} \left| Q^{-1}(t) \left[B\left(t, x^{m-1}(t)\right) - B\left(t, x^{m-2}(t)\right)\right] \right| +$$

$$+ \int_0^T \left| t^{1-\gamma} G(t, s) \right| (t-s)^{\alpha-1} \times$$

$$\times \left| f\left(s, x^{m-1}(s), \int_0^T \Theta\left(s, \theta, \max\left\{x^{m-1}(\tau) | \tau \in \left[h_1^{m-1} : | : h_2^{m-1}\right]\right\}\right) d\theta\right) - \right.$$

$$\left. - f\left(s, x^{m-2}(s), \int_0^T \Theta\left(s, \theta, \max\left\{x^{m-2}(\tau) | \tau \in \left[h_1^{m-2} : | : h_2^{m-2}\right]\right\}\right) d\theta\right) \right| ds +$$

$$+ \sum_{i=1}^{p} \left| t^{1-\gamma} G(t, t_i) \right| \left| F_i\left(x^{m-1}(t_i)\right) - F_i\left(x^{m-2}(t_i)\right) \right| \leqslant$$

$$\leqslant \frac{1}{\Gamma(\gamma)} \left| Q^{-1}(t) \right| M_4(t) \cdot \left| t^{1-\gamma} \big(x^{m-1}(t) - x^{m-2}(t) \big) \right| +$$

$$+ \int_0^T \left| t^{1-\gamma} G(t,s) \right| (t-s)^{\alpha-1} \Big[M_1(s) \cdot \left| s^{1-\gamma} \big(x^{m-1}(s) - x^{m-2}(s) \big) \right| + M_2(s) \int_0^T M_3(s,\theta) \times$$

$$\times \Big(\theta^{1-\gamma} \Big| \max \big\{ x^{m-1}(\tau) | \tau \in \big[h_1^{m-1} : | : h_2^{m-1} \big] \big\} -$$

$$- \max \big\{ x^{m-2}(\tau) | \tau \in \big[h_1^{m-1} : | : h_2^{m-1} \big] \big\} \Big| +$$

$$+ \theta^{1-\gamma} \Big| \max \big\{ x^{m-2}(\tau) | \tau \in h_1^{m-1} : | : h_2^{m-1} \big\} -$$

$$- \max \big\{ x^{m-2}(\tau) | \tau \in h_1^{m-2} : | : h_2^{m-2} \big\} \Big| \Big) \, d\theta \Big] ds +$$

$$+ \sum_{i=1}^{p} \left| t^{1-\gamma} G(t,t_i) \right| \cdot N_i \, \left| t_i^{1-\gamma} \big(x^{m-1}(t_i) - x^{m-2}(t_i) \big) \right| . \tag{15}$$

Taking into account that

$$\Big| \max \big\{ x^{m-1}(\tau) | \tau \in \big[h_1^{m-1} : | : h_2^{m-1} \big] \big\} - \max \big\{ x^{m-2}(\tau) | \tau \in \big[h_1^{m-1} : | : h_2^{m-1} \big] \big\} \Big| +$$

$$+ \Big| \max \big\{ x^{m-2}(\tau) | \tau \in \big[h_1^{m-1} : | : h_2^{m-1} \big] \big\} - \max \big\{ x^{m-2}(\tau) | \tau \in \big[h_1^{m-2} : | : h_2^{m-2} \big] \big\} \Big| \leqslant$$

$$\leqslant \left| x^{m-1}(t) - x^{m-2}(t) \right| + M_f \left[\left| h_1 \left(t, x^{m-1}(t) \right) - h_1 \left(t, x^{m-2}(t) \right) \right| + \right.$$

$$\left. + \left| h_2 \left(t, x^{m-1}(t) \right) - h_2 \left(t, x^{m-2}(t) \right) \right| \right] \leqslant$$

$$\leqslant \big[1 + M_f \big(M_5(t) + M_6(t) \big) \big] \left| x^{m-1}(t) - x^{m-2}(t) \right| ,$$

from (15) we easily obtain

$$\left| t^{1-\gamma} \left(x^m(t) - x^{m-1}(t) \right) \right| \leqslant \frac{1}{\Gamma(\gamma)} \left| Q^{-1}(t) \right| M_4(t) \cdot \left| t^{1-\gamma} \big(x^{m-1}(t) - x^{m-2}(t) \big) \right| +$$

$$+ \int_0^T \left| t^{1-\gamma} G(t,s) \right| (t-s)^{\alpha-1} \Big[M_1(s) \left| s^{1-\gamma} \big(x^{m-1}(s) - x^{m-2}(s) \big) \right| + M_2(s) \int_0^T M_3(s,\theta) \times$$

$$\times \big[1 + M_f \big(M_5(\theta) + M_6(\theta) \big) \big] \left| \theta^{1-\gamma} \big(x^{m-1}(\theta) - x^{m-2}(\theta) \big) \right| d\theta \Big] ds +$$

$$+ \sum_{i=1}^{p} \left| t^{1-\gamma} G(t,t_i) \right| \cdot N_i \cdot \left| t_i^{1-\gamma} \big(x^{m-1}(t_i) - x^{m-2}(t_i) \big) \right| .$$

Hence, by the aid of the introduced norm we get

$$\left\| t^{1-\gamma} \left(x^m(t) - x^{m-1}(t) \right) \right\|_{PC} \leqslant$$

$$\leqslant \frac{1}{\Gamma(\gamma)} \max_{0 \leqslant t \leqslant T} \left| Q^{-1}(t) \right| M_4(t) \, \left\| t^{1-\gamma} \big(x^{m-1}(t) - x^{m-2}(t) \big) \right\| +$$

$$+ \max_{0 \le t \le T} \int_0^T \left| t^{1-\gamma} G(t,s) \right| (t-s)^{\alpha-1} M_1(s) \, \left\| s^{1-\gamma} \big(x^{m-1}(s) - x^{m-2}(s) \big) \right\| \, ds +$$

$$+ \max_{0 \le t \le T} \int_0^T \left| t^{1-\gamma} G(t,s) \right| (t-s)^{\alpha-1} M_2(s) \int_0^T M_3(s,\theta) \times$$

$$\times \left[1 + M_f\big(M_5(\theta) + M_6(\theta)\big)\right] \left\| \theta^{1-\gamma}\big(x^{m-1}(\theta) - x^{m-2}(\theta)\big) \right\| d\theta ds +$$

$$+ \sum_{i=1}^{p} \left| t^{1-\gamma} G(t, t_i) \right| \cdot N_i \cdot \left\| t_i^{1-\gamma}\big(x^{m-1}(t_i) - x^{m-2}(t_i)\big) \right\| \leqslant$$

$$\leqslant \rho \cdot \left\| t^{1-\gamma}\big(x^{m-1}(t) - x^{m-2}(t)\big) \right\|_{PC}, \tag{16}$$

where $\rho = S_1 + S_2 + S_3 + S_4$.

According to the last condition of the theorem $\rho < 1$. Therefore, from the estimate (16) we have that the operator J on the right-hand side of the equation (12) is contracting. According to fixed point principle, taking into account estimates (14)–(16), we conclude that the operator J has a unique fixed point in the class $PC\left([0, T], \mathbb{R}^n\right)$. The theorem is proved. □

Consequently, nonlocal boundary value problem (1)–(3) has a unique solution $x(t) \in \mathbb{R}^n$ on $(0, T) \setminus \{t_i\}$, $i = 1, 2, \ldots, p$.

5. Conclusion

On the interval $(0, T) \setminus \{t_i\}$, $i = 1, 2, \ldots, p$ for $0 < \alpha \leq \gamma \leqslant 1$ is considered the fractional order system of nonlinear integro-differential equations (1) with nonlocal integral condition (2) and nonlinear impulsive condition (3). The problem (1)–(3) is reduced on the interval $(0, T) \setminus \{t_i\}$, $i = 1, 2, \ldots, p$ to the nonlinear system of functional integral equations (12). The system of functional integral equations (12) has terms of nonlinear functions in integral and non-integral forms. The theorem on existence and uniqueness of the solution of the nonlinear boundary value problem (1)–(3) is proved. In proving the theorem on one-valued solvability of the boundary value problem (1)–(3) is used the method of successive approximations in combination it with the method of compressing mapping.

References

1. **Samko S.G., Kilbas A.A., Marichev O.I.** *Fractional Integrals and Derivatives. Theory and Applications*. Philadelphia, Gordon and Breach, 1993.
2. **Mainardi F.** Fractional calculus: some basic problems in continuum and statistical mechanics. In: A. Carpinteri, F. Mainardi, F. (eds.) *Fractals and Fractional Calculus in Continuum Mechanics*. Wien, Springer, 1997.
3. **Area I., Batarfi H., Losada J., Nieto J.J., Shammakh W., Torres A.** On a fractional order Ebola epidemic model. *Advances in Difference Equations*, 2015, no. 278.
4. *Handbook of Fractional calculus with applications*. Vols. 1–8. J.A. Tenreiro Machado (ed.). Berlin, Boston, Walter de Gruyter, 2019.
5. **Kumar D., Baleanu D.** Editorial: fractional calculus and its applications in physics. *Frontiers in Physics*, 2019, vol. 7, no. 6.
6. **Sun H., Chang A., Zhang Y., Chen W.** A review on variable-order fractional differential equations: mathematical foundations, physical models, numerical methods and applications. *Fractional Calculus and Applied Analysis*, 2019, vol. 22, no. 1, pp. 27–59.
7. **Hussain A., Baleanu D., Adeel M.** Existence of solution and stability for the fractional order novel coronavirus (nCoV-2019) model. *Advances in Difference Equations*, 2020, no. 384.
8. **Patnaik S., Hollkamp J.P., Semperlotti F.** Applications of variable-order fractional operators: a review. *Proceedings of the Royal Society A*, 2020, vol. 476, no. 2234.

9. **Ullah S., Khan M.A., Farooq M., Hammouch Z., Baleanu D.** A fractional model for the dynamics of tuberculosis infection using Caputo — Fabrizio derivative. *Discrete and Continuous Dynamical Systems. Series S*, 2020, vol. 13, no. 3, pp. 975–993.
10. **Abdullaev O.Kh., Islomov B.I.** Gellerstedt type problem for the loaded parabolic-hyperbolic type equation with Caputo and Erdelyi — Kober operators of fractional order. *Russian Mathematics*, 2020, vol. 64, no. 10, pp. 29–42.
11. **Karimov E.T.** Frankl-type problem for a mixed type equation with the Caputo fractional derivative. *Lobachevskii Journal of Mathematics*, 2020, vol. 41, no. 9, pp. 1829–1836.
12. **Yuldashev T.K., Kadirkulov B.J.** Nonlocal problem for a mixed type fourth-order differential equation with Hilfer fractional operator. *Ural Mathematical Journal*, 2020, vol. 6, no. 1, pp. 153–167.
13. **Yuldashev T.K., Karimov E.T.** Inverse problem for a mixed type integro-differential equation with fractional order Caputo operators and spectral parameters. *Axioms*, 2020, vol. 9, no. 121.
14. **Fedorov V.E., Plekhanova M.V., Izhberdeeva E.M.** Initial value problems of linear equations with the Dzhrbashyan — Nersesyan derivative in Banach spaces. *Symmetry*, 2021, vol. 13, no. 1058.
15. **Fedorov V. E., Turov M.M.** The defect of a Cauchy type problem for linear equations with several Riemann — Liouville derivatives. *Siberian Math. Journal*, 2021, vol. 62, no. 5, pp. 925–942.
16. **Volkova A.R., Izhberdeeva E.M., Fedorov V.E.** Initial value problem for linear equations with composition of fractional derivatives. *Chelyabinsk Physical and Mathematical Journal*, 2021, vol. 6, no. 3, pp. 269–277.
17. **Yuldashev T.K., Abdullaev O.Kh.** Unique solvability of a boundary value problem for a loaded fractional parabolic-hyperbolic equation with nonlinear terms. *Lobachevskii Journal of Mathematics*, 2021, vol. 42, no. 5, pp. 1113–1123.
18. **Yuldashev T.K., Kadirkulov B.J.** Inverse boundary value problem for a fractional differential equations of mixed type with integral redefinition conditions. *Lobachevskii Journal of Mathematics*, 2021, vol. 42, no. 3, pp. 649–662.
19. **Volkova A.R., Fedorov V.E., Gordievskikh D.M.** On solvability of some classes of equations with Hilfer derivatives in Banach spaces. *Chelyabinsk Physical and Mathematical Journal*, 2022, vol. 7, no. 1, pp. 11–19.
20. **Halanay A., Wexler D.** *Teoria calitativa a sistemelor cu impulsuri*. Bucuresti, Editura Academiei Republicii Socialiste Romania, 1968.
21. **Lakshmikantham V., Bainov D.D., Simeonov P.S.** *Theory of Impulsive Differential Equations*. Singapore, World Scientific, 1989.
22. **Samoilenko A.M., Perestyuk N.A.** *Impulsive differential equations*. Singapore, World Scientific, 1995.
23. **Benchohra M., Henderson J., Ntouyas S.K.** *Impulsive Differential Equations and Inclusions*. New York, Hindawi Publ., 2006.
24. **Boichuk A.A., Samoilenko A.M.** *Generalized Inverse Operators and Fredholm Boundary Value Problems*. Berlin, Boston, Walter de Gruyter, 2016.
25. **Yuldashev T.K., Fayziev A.K.** On a nonlinear impulsive differential equations with maxima. *Bulletin of the Institute of Mathematics*, 2021, vol. 4, no. 6, pp. 42–49.
26. **Yuldashev T.K.** Periodic solutions for an impulsive system of nonlinear differential equations with maxima. *Nanosystems: Physics. Chemistry. Mathematics*, 2022, vol. 13, no. 2, pp. 135–141.
27. **Yuldashev T.K., Fayziev A.K.** On a nonlinear impulsive system of integro-differential equations with degenerate kernel and maxima. *Nanosystems: Physics. Chemistry. Mathematics*, 2022, vol. 13, no. 1, pp. 36–44.

28. **Yuldashev T.K.** Nonlocal mixed-value problem for a Boussinesq-type integrodifferential equation with degenerate kernel. *Ukrainian Mathematical Journal*, 2016, vol. 68, no. 8, pp. 1278–1296.
29. **Yuldashev T.K.** Determination of the coefficient and boundary regime in boundary value problem for integro-differential equation with degenerate kernel. *Lobachevskii Journal of Mathematics*, 2017, vol. 38, no. 3, pp. 547–553.
30. **Yuldashev T.K.** On the solvability of a boundary value problem for the ordinary Fredholm integrodifferential equation with a degenerate kernel. *Computational Mathematics and Mathematical Physics*, 2019, vol. 59, no. 2, pp. 241–252.
31. **Yuldashev T.K.** Spectral features of the solving of a Fredholm homogeneous integro-differential equation with integral conditions and reflecting deviation. *Lobachevskii Journal of Mathematics*, 2019, vol. 40, no. 12, pp. 2116–2123.
32. **Yuldashev T.K.** On inverse boundary value problem for a Fredholm integro-differential equation with degenerate kernel and spectral parameter. *Lobachevskii Journal of Mathematics*, 2019, vol. 40, no. 2, pp. 230–239.
33. **Assanova A.** An integral-boundary value problem for a partial differential equation of second order. *Turkish Journal of Mathematics*, 2019, vol. 43, no. 4, pp. 1967–1978.
34. **Assanova A.T.** On the solvability of nonlocal problem for the system of Sobolev-type differential equations with integral condition. *Georgian Mathematical Journal*, 2021, vol. 28, no. 1, pp. 49–57.
35. **Assanova A.T., Imanchiyev A.E., Kadirbayeva Zh.M.** A nonlocal problem for loaded partial differential equations of fourth order. *Bulletin of the Karaganda University. Ser. Mathematics*, 2020, vol. 97, no. 1, pp. 6–16.
36. **Assanova A.T., Tokmurzin Z.S.** A nonlocal multipoint problem for a system of fourth-order partial differential equations. *Eurasian Mathematical Journal*, 2020, vol. 11, no. 3, pp. 8–20.
37. **Minglibayeva A.B., Assanova A.T.** An existence of an isolated solution to nonlinear two-point boundary value problem with parameter. *Lobachevskii Journal of Mathematics*, 2021, vol. 42, no. 3, pp. 587–597.
38. **Anguraj A., Arjunan M.M.** Existence and uniqueness of mild and classical solutions of impulsive evolution equations. *Electronic Journal of Differential Equations*, 2005, no. 111.
39. **Bin L., Xinzhi L., Xiaoxin L.** Robust global exponential stability of uncertain impulsive systems. *Acta Mathematika Scientia*, 2005, vol. 25, no. 1, pp. 161–169.
40. **Li M., Han M.** Existence for neutral impulsive functional differential equations with nonlocal conditions. *Indagationes Mathematicae*, 2009, vol. 20, no. 3, pp. 435–451.
41. **Ji Sh., Wen Sh.** Nonlocal Cauchy problem for impulsive differential equations in Banach spaces. *International Journal of Nonlinear Science*, 2010, vol. 10, no. 1, pp. 88–95.
42. **Sharifov Y.A.** Conditions optimality in problems control with systems impulsive differential equations with nonlocal boundary conditions. *Ukrainain Mathematical Journal*, 2012, vol. 64, no. 6, pp. 836–847.
43. **Ashyralyev A., Sharifov Y.A.** Existence and uniqueness of solutions for nonlinear impulsive differential equations with two-point and integral boundary conditions. *Advances in Difference Equations*, 2013, vol. 2013, no. 173.
44. **Gao Zh., Yang L., Liu G.** Existence and uniqueness of solution of impulsive fractional integro-differential equations with nonlocal conditions. *Applied Mathematics*, 2013, vol. 4, pp. 859–863.
45. **Sharifov Ya.A.** Optimal control for systems with impulsive actions under nonlocal boundary conditions. *Russian Mathematics*, 2013, vol. 57, no. 2, pp. 65–72.
46. **Agarwal R.P., Benchohra M., Salimani B.A.** Existence results for differential equations with fractional order and impulses. *Memoirs on Differential Equations and Mathematical Physics*, 2008, vol. 44, pp. 1–21.

47. **Benchohra M., Salimani B.A.** Existence and uniqueness of solutions to impulsive fractional differential equations. *Electronic Journal of Differential Equations*, 2009, vol. 2009, no. 10.
48. **Feckan M., Zhong Y., Wang J.** On the concept and existence of solutions for impulsive fractional differential equations. *Communications in Nonlinear Science and Numerical Simulation*, 2012, vol. 17, no. 7, pp. 3050–3060.
49. **Nadeem M., Dabas J.** Impulsive stochastic fractional order integro-differential equations with infinite delay. *Journal of Nonlinear Evolution Equations and Applications*, 2019, vol. 2017, no. 8, pp. 109–121.
50. **Yuldashev T.K.** On a nonlocal problem for impulsive differential equations with mixed maxima. *Vestnik KRAUNTs. Fiziko-matematicheskiye nauki* [Bulletin of KRAUNTs. Physical and Mathematical Sciences], 2022, vol. 38, no. 1, pp. 40–53.
51. **Yuldashev T.K., Saburov Kh.Kh., Abduvahobov T.A.** Nonlocal problem for a nonlinear system of fractional order impulsive integro-differential equationswith maxima. *Chelyabinsk Physical and Mathematical Journal*, 2022, vol. 7, no. 1, pp. 113–122.

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НЕЛИНЕЙНЫЕ СИСТЕМЫ ИНТЕГРО-ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ С ИМПУЛЬСНЫМИ ВОЗДЕЙСТВИЯМИ, ДРОБНЫМИ ОПЕРАТОРАМИ ХИЛФЕРА И СМЕШАННЫМИ МАКСИМУМАМИ

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Исследуется нелокальная краевая задача для системы обыкновенных интегро-дифференциальных уравнений с импульсными воздействиями, нелинейными смешанными максимумами и дробным оператором Хилфера. Граничное условие задаётся в нелинейной интегральной форме. Задача сводится к нелинейной системе функциональных интегральных уравнений. Система функциональных интегральных уравнений имеет слагаемые в виде нелинейных функций в интегральной и неинтегральной формах. При доказательстве однозначной разрешимости краевой задачи используется метод последовательных приближений в сочетании с методом сжимающих отображений. Доказаны существование и единственность решения нелинейной краевой задачи.

Ключевые слова: *интегро-дифференциальное уравнение с импульсными воздействиями, дробный оператор Хилфера, нелокальное граничное условие, смешанные максимумы, однозначная разрешимость.*

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