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INFLUENCE OF GRAPHENE-BASED HYPERBOLIC METASURFACE PARAMETERS ON THE SECOND HARMONIC GENERATION

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We have investigated the influence of graphene-based hyperbolic metasurface parameters such as the structure periodicity, the ribbon's width, the graphene chemical potential and the frequency of exciting light on the second harmonic generation phenomenon. The investigation showed that this process may be observed just for a fixed angle of the surface plasmon-polaritons propagation. Taking into account the possibility to control graphene properties (i. e., its chemical potential, or Fermi level) by the gate voltage, for example, our results may became the basis for non-linear optical reconfigurable devices.

Keywords: *graphene, hyperbolic metasurface, surface plasmon-polaritons, second harmonic generation.*

Introduction

Spatial nanostructuring of graphene offers a unique platform for the surface plasmon-polariton (SPP) manipulation, known as plasmonic metasurfaces. Metasurfaces are a two-dimensional case of metamaterials – artificial structures whose properties allow the observation of unique, even unnatural interactions with electromagnetic waves [1; 2]. But as three-dimensional structures metamaterials suffer from huge propagation losses [3; 4]. Metamaterials have the potential to mitigate this problem [5–7]. Moreover, strongly anisotropic metasurfaces have shown the ability to control the radiation of localized near-field sources at terahertz and near infrared frequencies and guide the resulting surface waves in determined directions along the surface [8–13]. In that regard, hyperbolic metasurfaces are uniaxial structures with the extreme anisotropy, which is defined by the conductivity/impedance tensor [8; 11; 12]. Hyperbolic metasurfaces may demonstrate a not usual elliptic but hyperbolic dispersion for the wave propagation that makes metasurface to act as a dielectric in one direction and as a metal in orthogonal direction [14; 15].

The nonlinearity of graphene-based hyperbolic metasurface allows to observe the phenomenon of the second harmonic generation (SHG). It is a nonlinear optical process where two photons at the same frequency are converted into a single photon with

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twice the frequency and the energy. Here we investigate the influence of graphene-based hyperbolic metasurface parameters such as the structure periodicity, the ribbon's width, the graphene chemical potential and the frequency of exciting light on this phenomenon.

1. Wave propagation in graphene-based hyperbolic metasurfaces

The behavior of a hyperbolic metasurface can be characterized by the general conductivity tensor

$$\bar{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}.$$

Its elements are generally complex. The conductivity tensor must be diagonal in our coordinate system. Off-diagonal components can arise, if the subwavelength meta-atoms are nonsymmetrical relatively the coordinate system [13; 16] or magneto-optical effects take place [13]. Also, we assume a time convention as $e^{-i\omega t}$.

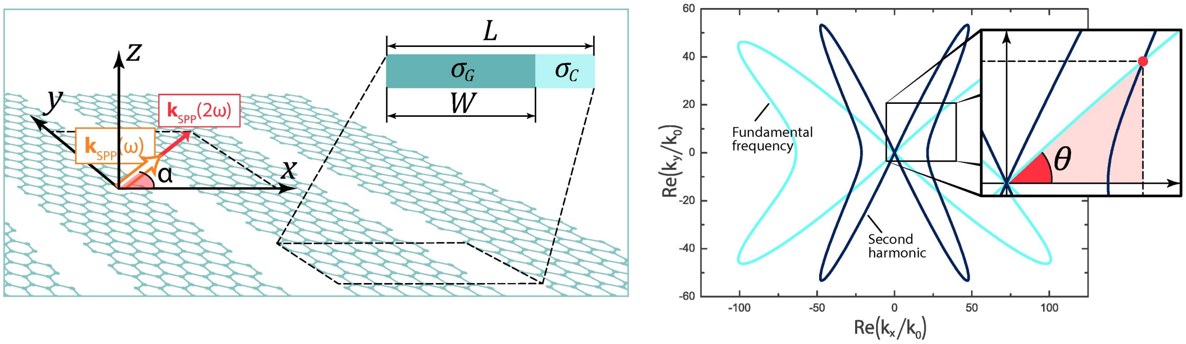

Fig. 1. A schematic model of the graphene-based hyperbolic metasurface (a) and the isofrequency contour with the phase matching angle (b)

Fig. 1 shows a schematic model of the graphene-based hyperbolic metasurface investigated in this work.

The metasurface consists of an array of the densely-packed graphene strips with a period L and the strips width W. Taking into account that the condition $L \ll \lambda_{SPP}$ is satisfied, where λ_{SPP} is the SPP wavelength, the in-plane effective conductivity tensor of this graphene-based metasurface can be analytically derived using effective medium theory as in [11]

$$\sigma_{xx}^{\mathrm{eff}} = \frac{L\sigma_G\sigma_C}{W\sigma_C + G\sigma_G}, \quad \sigma_{yy}^{\mathrm{eff}} = \sigma_G\frac{W}{L},$$

where G is the separation distance between two consecutive strips, σ_G is the graphene conductivity and σ_C is an effective conductivity related to the near-field coupling between adjacent strips, obtained using the electrostatic approach. The conductivity of homogeneous graphene at a frequency $\omega \approx 2\pi f$, where f is a linear frequency, is as follows:

$$\sigma_G(\omega) = \sigma_{\mathrm{intra}}(\omega) + \sigma_{\mathrm{inter}}(\omega),$$
$$\sigma_{\mathrm{intra}}(\omega) = \frac{2ie^2k_BT}{\pi\hbar\left(\omega + i\tau^{-1}\right)} \ln\left[2\cosh\left(\frac{\mu_{\mathrm{ch}}}{2k_BT}\right)\right],$$
$$\sigma_{\mathrm{inter}}(\omega) = \frac{e^2}{4\hbar}\left[\frac{1}{2} + \frac{1}{\pi}\arctan\left(\frac{\hbar\omega - 2\mu_{\mathrm{ch}}}{2k_BT}\right) - \frac{1}{2\pi}\ln\frac{\left(\hbar\omega + 2\mu_{\mathrm{ch}}\right)^2}{\left(\hbar\omega - 2\mu_{\mathrm{ch}}\right)^2 + \left(2k_BT\right)^2}\right],$$

where T is a given temperature, μ_{ch} is a graphene chemical potential. $\sigma_{\mathrm{intra}}(\omega)$ corresponds to the intraband electron-phonon scattering process, and $\sigma_{\mathrm{inter}}(\omega)$ meets the direct interband electron transitions and is leading about the absorption edge $\hbar\omega \approx 2\mu_{\mathrm{ch}}$ [17].

An effective conductivity σ_C has the following form [10]:

$$\sigma_C(\omega) = -i\omega\varepsilon_0\varepsilon_{\mathrm{eff}}\left(\frac{L}{\pi}\right)\ln\left[\csc\left(\frac{\pi G}{2L}\right)\right],$$

where $\varepsilon_{\mathrm{eff}}$ is the effective permittivity of the media that embed the ribbons.

The dispersion relation of SPPs propagating along our hyperbolic metasurface is as follows [10]:

$$\left(2\frac{k_z}{k_0} + \eta_0{\sigma'}_{yy}\right)\left(2\frac{k_0}{k_z} + \eta_0{\sigma'}_{xx}\right) - \eta_0^2\sigma'_{xy}\sigma'_{yx} = 0, \tag{1}$$

where η_0 is the free-space impedance, σ'_{xx}, σ'_{xy}, σ'_{yx} and σ'_{yy} denote elements of the rotated conductivity tensor

$$\bar{\sigma}' = \begin{pmatrix} {\sigma'}_{xx} & {\sigma'}_{xy} \\ {\sigma'}_{yx} & {\sigma'}_{yy} \end{pmatrix} = \bar{\mathbf{R}}^T\bar{\sigma}\,\bar{\mathbf{R}}. \tag{2}$$

Here $\bar{\mathbf{R}}$ and $\bar{\mathbf{R}}^T$ are standard rotation matrixes:

$$\begin{aligned}\bar{\mathbf{R}} &= \begin{pmatrix} \cos(\varphi) & -\sin(\varphi) \\ \sin(\varphi) & \cos(\varphi) \end{pmatrix} = \frac{1}{k_p}\begin{pmatrix} k_x & -k_y \\ k_y & k_x \end{pmatrix},\\ \bar{\mathbf{R}}^T &= \begin{pmatrix} \cos(\varphi) & \sin(\varphi) \\ -\sin(\varphi) & \cos(\varphi) \end{pmatrix} = \frac{1}{k_p}\begin{pmatrix} k_x & k_y \\ -k_y & k_x \end{pmatrix},\end{aligned}$$

with $k_x = k_p\cos(\varphi)$ and $k_y = k_p\sin(\varphi)$.

In order to solve equation (1), we fix the SPPs direction of propagation along one particular angle within a coordinate system, for instance the x axis (i. e., $k_y = 0$), and then physically rotate the metasurface an angle φ using equation (2) to estimate the features of SPPs propagating along that direction. Taking this approach, we get two possible solutions for the transverse wavenumber k_z at any direction of φ:

$$\begin{aligned}k_z^{\pm}(\varphi) = \tfrac{k_0}{2{\sigma'}_{xx}}\Big[&-\left(\tfrac{2}{\eta_0} + \tfrac{\eta_0}{2}\left({\sigma'}_{xx}{\sigma'}_{xy} - {\sigma'}_{xy}{\sigma'}_{yy}\right)\right) \pm \\ &\pm\sqrt{\left(\tfrac{2}{\eta_0} + \tfrac{\eta_0}{2}\left({\sigma'}_{xx}{\sigma'}_{yy} - {\sigma'}_{xy}{\sigma'}_{yx}\right)\right)^2 - 4{\sigma'}_{xx}{\sigma'}_{yy}}\Big].\end{aligned}$$

SPPs are supported by an infinitesimally-thin surface only when the transverse wavenumbers are evanescent, i. e., $\mathrm{Im}[k_z(\varphi)] > 0$ [18]. The complete solution of the dispersion relation (1) evaluates to $k_p = \sqrt{k_0^2 - k_z^2(\varphi)}$. This solution is used to calculate the expressions for the in-plane k-vector components and plot isofrequency contours.

2. The second harmonic generation

In order to obtain the second harmonic in metasurface, the phase matching condition must be satisfied:

$$2k_{SPP}(\omega) = k_{SPP}(2\omega),$$

that can be submitted as

$$\mathrm{Re}\left(\frac{2k_p(\omega)}{k_0}\right) = \mathrm{Re}\left(\frac{k_p(2\omega)}{k_0}\right).$$

We must solve this equation for the certain angle θ. That angle θ is the required phase matching angle. Figure 1(b) shows that angle on isofrequency contours. If the SPP propagating angle α is equal to the phase matching angle, it will be possible to observe the second harmonic generation.

For all calculations we assumed the following parameters: $T = 300$ K, $L = 50$ nm. Fig. 2 shows the obtained dependence of the phase matching angle θ on the ribbon width W. We observed this dependence at different frequency ω (Fig. 2, a) and chemical potential μ_{ch} (Fig. 2, b). The plot has a minimum in a particular point, which depends on the metasurface and incidence radiation properties. Also it is clear that increasing of the ribbon width increases the phase propagation angle. Besides, the frequency of incident radiation has the same influence as it is shown in Fig. 3, a. Conversely, the phase propagation angle is decreasing with increasing the chemical potential (Fig. 3, b).

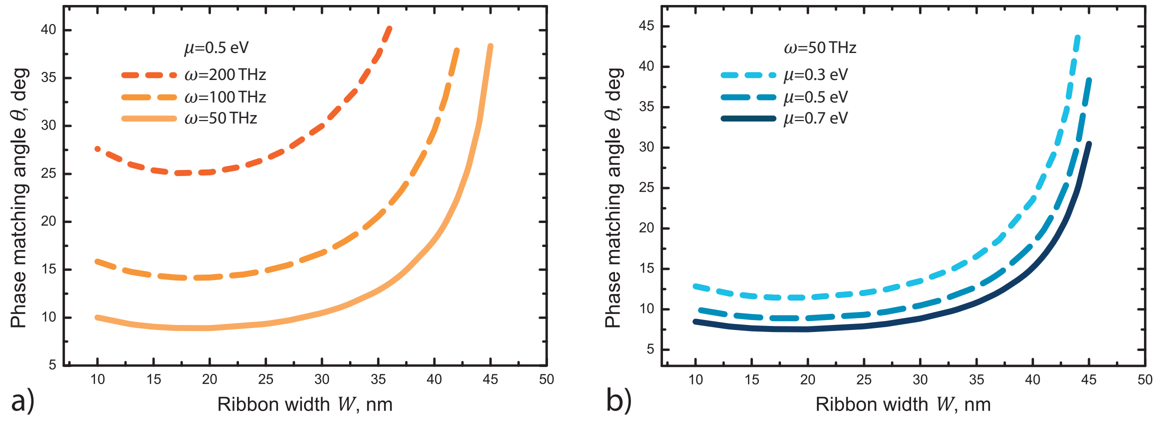

Fig. 2. Dependence of the phase matching angle θ on ribbon width W at different frequency ω (a) and chemical potential μ (b)

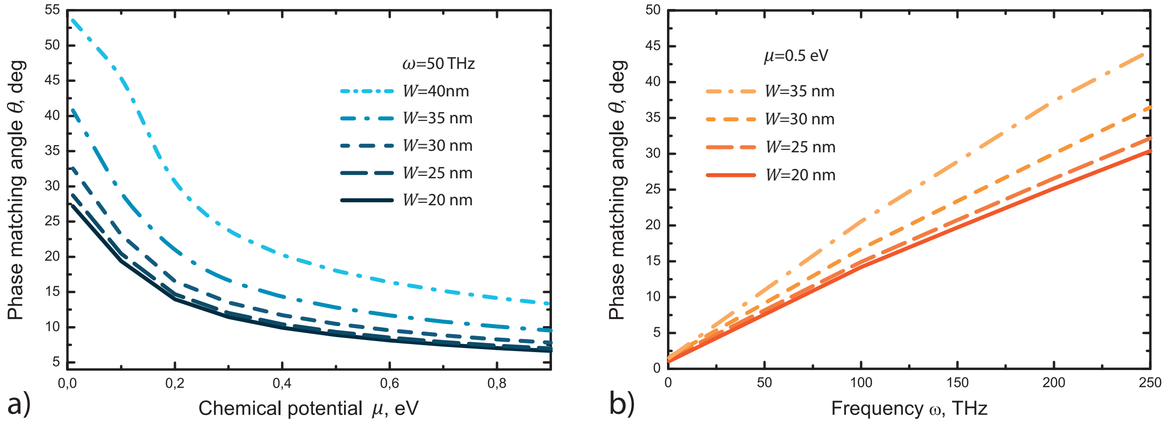

Fig. 3. Dependence of the phase matching angle θ on chemical potential μ (a) and frequency ω (b) at different ribbon width W

Conclusions

We have investigated the possibility of the second harmonic generation process in the graphene-based metasurface from SPPs at the fundamental frequency to SPP at the doubled frequency. The investigation showed that this process may be observed just for a fixed angle of the SPPs propagation. This angle depends on the frequency of fundamental SPPs, metasurface geometrical parameters and graphene properties. Taking into account the possibility to control graphene properties (i. e., its chemical potential, or Fermi level) by the gate voltage, for example, our results may became the basis for non-linear optical reconfigurable devices.

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ВЛИЯНИЕ ПАРАМЕТРОВ МЕТАПОВЕРХНОСТИ НА ОСНОВЕ ГРАФЕНА НА ГЕНЕРАЦИЮ ВТОРОЙ ГАРМОНИКИ

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Исследовано влияние на генерацию второй гармоники параметров гиперболической метаповерхности на основе графена: ее периодичности, ширины графеновых лент, химического потенциала графена и частоты падающего света. Исследование показало, что такой процесс может наблюдаться только для определённых углов распространения поверхностных плазмон-поляритонов. Если принять во внимание возможность управления свойстваим графена (его химическим потенциалом, или энергией Ферми) при помощи, например, электрического напряжения, наши результаты могут стать основой для нелинейных оптических перестраиваимых устройств.

Ключевые слова: *графен, гиперболические метаповерхности, поверхностные плазмон-поляритоны, генерация второй гармоники.*

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