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MIXED PROBLEM FOR A NONLINEAR IMPULSIVE DIFFERENTIAL EQUATION OF PARABOLIC TYPE

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In this paper, we consider a nonlinear impulsive parabolic type partial differential equation with nonlinear impulsive conditions. Dirichlet type boundary value conditions with respect to spatial variable is used, and eigenvalues and eigenfunctions of the spectral problem are founded. The Fourier method of the separation of variables is applied. A countable system of nonlinear functional equations is obtained with respect to the Fourier coefficients of the unknown function. A theorem on a unique solvability of the countable system of nonlinear functional equations is proved by the method of successive approximations. A criteria of uniqueness and existence of a solution for the nonlinear impulsive mixed problem is obtained. A solution of the mixed problem is derived in the form of the Fourier series. The absolute and uniform convergence of the Fourier series is proved.

Keywords: *mixed problem, nonlinear parabolic equation, nonlinear impulsive conditions, involution, unique solvability.*

1. Formulation of the problem statement

Partial differential and integro-differential equations of parabolic type are investigated widely by large number of scientists and have different applications in sciences and technology (see, for example [1–14]). Differential and integro-differential equations with impulse effects have applications in biological, chemical and physical sciences, ecology, biotechnology, industrial robotic, pharmacokinetics, optimal control, etc. [15–21]. A lot of publications are devoted to study differential equations with impulsive effects, describing many natural and technical processes (see, for example [22–38]). In [39–43], the questions of existence and uniqueness of periodic solutions of differential and integro-differential equations were studied. In [44–46] Whitham type partial differential equations of first order with impulsive effects were investigated.

In this paper, we study the solvability of the impulsive mixed problem for a nonlinear parabolic type differential equation with involution and nonlinear impulsive conditions. In the domain $\Omega = \{t \in (0,T),\, t \neq t_m,\, 0 < t_m < T,\, m = 1,2,\ldots,p,\, x \in (-1,1)\}$ we consider the following nonlinear partial differential equation

$$U_t(t,x) - U_{xx}(t,x) - \varepsilon\, U_{xx}(t,-x) = f\left(t,x,\int\limits_{-1}^{1} K(y)\, U(t,y)dy\right) \tag{1}$$

with Dirichlet boundary value conditions on spatial variable x

$$U(t,-1) = U(t,1) = 0, \qquad 0 \leqslant t \leqslant T, \tag{2}$$

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and initial value condition on time variable t

$$U(0,x)=\varphi(x), \quad -1\leqslant x\leqslant 1, \tag{3}$$

where T is given positive number, ε is parameter, $|\varepsilon|<1$, $\varphi(x)$ is given function on the segment $[-1,1]$. We assume that for the initial value function $\varphi(x)\in C[-1,1]$ the following conditions are fulfilled $\varphi(-1)=\varphi(1)=0$. We also suppose that $f(t,x,\cdot)\in C(\Omega\times\mathbb{R})$ and $f(t,-1,\cdot)=f(t,1,\cdot)=0$.

Since equation (1) is impulsive, we use also nonlinear impulsive conditions

$$U\left(t_m^+,x\right)-U\left(t_m^-,x\right)=I_m\left(x,\int\limits_{-1}^{1}G(y)U\left(t_m,y\right)dy\right),\ m=1,2,\ldots,p, \tag{4}$$

where $\int\limits_{-1}^{1}|G(x)|dx<\infty$, $0<t_1<\cdots<t_p<T<\infty$, $U\left(t_m^+,x\right)=\lim\limits_{\nu\to 0^+}U\left(t_m+\zeta,x\right)$, $U\left(t_m^-,x\right)=\lim\limits_{\nu\to 0^-}U\left(t_m-\zeta,x\right)$ are right-hand side and left-hand side limits of function $U(t,x)$ at the points $t=t_m$, respectively, $I_m(-1,\cdot)=I_m(1,\cdot)=0$.

2. Formal solution of the mixed problem

First, consider the homogeneous partial differential equation

$$U_t(t,x)-U_{xx}(t,x)-\varepsilon U_{xx}(t,-x)=0 \tag{5}$$

with boundary value conditions of the Dirichlet type

$$U(t,-1)=U(t,1)=0,\ \ 0\leqslant t\leqslant T. \tag{6}$$

Problem (5), (6) will be solved by the method of separation of variables: $U(t,x)=u(t)\,\vartheta(x)$. Then, using this method of separation of variables, from problem (5), (6) we arrive at the following spectral problem for an ordinary differential equation

$$\vartheta''(x)+\varepsilon\vartheta''(-x)+\lambda\vartheta(x)=0 \tag{7}$$

with boundary value conditions

$$\vartheta(-1)=0,\quad \vartheta(1)=0. \tag{8}$$

It is obvious that for the case of even eigenfunctions, equation (7) takes the form

$$(1+\varepsilon)\vartheta_1''(x)+\lambda_1\vartheta_1(x)=0. \tag{9}$$

Solving differential equation (9) with conditions (8), we find the eigenvalues

$$\lambda_{1,n}=(1+\varepsilon)\,\pi^2(n+0.5)^2 \tag{10}$$

and eigenfunctions of problem (7), (8):

$$\vartheta_{1,n}(x)=\cos\pi(n+0.5)x,\quad n\in\mathbb{N}. \tag{11}$$

In the case of odd eigenfunctions, equation (7) takes another form

$$(1-\varepsilon)\vartheta_2''(x)+\lambda_2\vartheta_2(x)=0. \tag{12}$$

Solving differential equation (12) with spectral (zero) conditions (8), we find the eigenvalues and the corresponding eigenfunctions of problem (12), (8)

$$\lambda_{2,n} = (1-\varepsilon)\,\pi^2 n^2, \quad |\,\varepsilon\,| < 1, \tag{13}$$

$$\vartheta_{2,n}(x) = \sin \pi n\, x, \quad n \in \mathbb{N}. \tag{14}$$

Note that the eigenfunctions $\vartheta_{i,n}(x)$ $(i = 1, 2)$ determined by (11) and (14) form a complete system of orthonormal eigenfunctions in the space $L_2[-1, 1]$. Therefore, we seek nontrivial solutions to the nonhomogeneous partial differential equation (1) in the forms $U(t, x) = U_1(t, x) + U_2(t, x)$ of the Fourier series

$$U_i(t, x) = \sum_{n=1}^{\infty} u_{i,n}(t)\,\vartheta_{i,n}(x), \quad i = 1, 2, \tag{15}$$

where $U_1(t, x)$ and $U_2(t, x)$ satisfy the given differential equation (1)

$$U_{it}(t, x) - U_{ixx}(t, x) - \varepsilon\, U_{ixx}(t, -x) = f_i\bigg(t, x, \int\limits_{-1}^{1} K(y)\, U_i(t, y)dy\bigg),$$

$$u_{i,n}(t) = \int\limits_{-1}^{1} U_i(t, x)\,\vartheta_{i,n}(x)dx, \quad i = 1, 2. \tag{16}$$

We also suppose that the function $f(t, x, \cdot) = f_1(t, x, \cdot) + f_2(t, x, \cdot) \in C(\Omega \times \mathbb{R})$ expands to the Fourier series by eigenfunctions $\vartheta_{i,n}(x)$:

$$f_i\bigg(t, x, \int\limits_{-1}^{1} K(y)\, U_i(t, y)dy\bigg) = \sum_{n=1}^{\infty} f_{i,n}(t, \cdot)\,\vartheta_{i,n}(x), \quad i = 1, 2,$$

where

$$f_{i,n}(t, \cdot) = \int\limits_{-1}^{1} f_i\bigg(t, x, \int\limits_{-1}^{1} K(y)\, U_i(t, y)dy\bigg)\,\vartheta_{i,n}(x)\,dx, \quad i = 1, 2. \tag{17}$$

By the aid of Fourier series (15), from the impulsive conditions (4) we obtain

$$\sum_{n=1}^{\infty} u_{i,n}(t_m^+)\,\vartheta_{i,n}(x) - \sum_{n=1}^{\infty} u_{i,n}(t_m^-)\,\vartheta_{i,n}(x) = \sum_{n=1}^{\infty} I_{m,i,n}(\cdot)\,\vartheta_{i,n}(x), \quad m = 1, 2, \ldots, p,$$

where

$$I_{m,i,n}(\cdot) = \int\limits_{-1}^{1} I_{m,i}\bigg(x, \int\limits_{-1}^{1} G(y) \sum_{j=1}^{\infty} u_{i,j}(t_m)\,\vartheta_{i,j}(y)dy\bigg)\,\vartheta_{i,n}(x)dx, \quad i = 1, 2. \tag{18}$$

Hence, by the aid of inner production

$$(\vartheta_{i,n}(x), \vartheta_{i,j}(x)) = \begin{cases} 1, & n = j, \\ 0, & n \neq j, \end{cases}$$

we derive that

$$u_{i,n}(t_m^+) - u_{i,n}(t_m^-) = I_{m,i,n}(\cdot), \quad m = 1, 2, \ldots, p. \tag{19}$$

Substituting the Fourier series (15) into the given differential equation (1), we obtain a countable system of the first order ordinary differential equations

$$u'_{i,n}(t) + \lambda_{i,n}\, u_{i,n}(t) = f_{i,n}(t,\cdot), \tag{20}$$

where $\lambda_{i,n}$ are eigenvalues determined by (10) and (13). The countable system of first order ordinary differential equations (20) we rewrite as

$$\left(e^{\lambda_{i,n}t} u_{i,n}(t)\right)' = e^{\lambda_{i,n}t} f_{i,n}(t,\cdot), \quad i = 1,2. \tag{21}$$

By integration of (21) on the intervals $(0,t_1]$, $(t_1,t_2]$, $\ldots$, $(t_p,t_{p+1}]$, we obtain

$$\int\limits_0^{t_1} e^{\lambda_{i,n}s} f_{i,n}(s,\cdot)ds = \int\limits_0^{t_1} \left(e^{\lambda_{i,n}s} u_{i,n}(s)\right)' ds =$$

$$= e^{\lambda_{i,n}t_1^-} u_{i,n}(t_1^-) - e^{\lambda_{i,n}0^+} u_{i,n}(0^+) = e^{\lambda_{i,n}t_1} u_{i,n}(t_1) - u_{i,n}(0), \quad i = 1,2, \quad t \in (0,t_1],$$

$$\int\limits_{t_1}^{t_2} e^{\lambda_{i,n}s} f_{i,n}(s,\cdot)ds = \int\limits_{t_1}^{t_2} \left(e^{\lambda_{i,n}s} u_{i,n}(s)\right)' ds =$$

$$= e^{\lambda_{i,n}t_2^-} u_{i,n}(t_2^-) - e^{\lambda_{i,n}t_1^+} u_{i,n}(t_1^+) = e^{\lambda_{i,n}t_2} u_{i,n}(t_2) - e^{\lambda_{i,n}t_1} u_{i,n}(t_1^+), \quad t \in (t_1,t_2],$$

$$\ldots, \int\limits_{t_p}^{t_{p+1}} e^{\lambda_{i,n}s} f_{i,n}(s,\cdot)ds = \int\limits_{t_p}^{t_{p+1}} \left(e^{\lambda_{i,n}s} u_{i,n}(s)\right)' ds =$$

$$= e^{\lambda_{i,n}t_{p+1}^-} u_{i,n}(t_{p+1}^-) - e^{\lambda_{i,n}t_p^+} u_{i,n}(t_p^+) = e^{\lambda_{i,n}t_{p+1}} u_{i,n}(t) - e^{\lambda_{i,n}t_p} u_{i,n}(t_p^+), \quad t \in (t_p,t_{p+1}].$$

Here we took into account that $u_{i,n}(0^+) = u_{i,n}(0)$, $u_{i,n}(t_{p+1}^-) = u_{i,n}(t)$. So, taking the impulsive conditions (19) into account, on the interval $(0,T]$ we have

$$\int\limits_0^t e^{\lambda_{i,n}s} f_{i,n}(s,\cdot)ds = e^{\lambda_{i,n}t_1} u_{i,n}(t_1) - u_{i,n}(0) + e^{\lambda_{i,n}t_2} u_{i,n}(t_2) - e^{\lambda_{i,n}t_1} u_{i,n}(t_1^+) +$$

$$+ e^{\lambda_{i,n}t_3} u_{i,n}(t_3) - e^{\lambda_{i,n}t_2} u_{i,n}(t_2^+) + \cdots + e^{\lambda_{i,n}t_{p+1}} u_{i,n}(t) - e^{\lambda_{i,n}t_p} u_{i,n}(t_p^+) =$$

$$= -u_{i,n}(0) - e^{\lambda_{i,n}t_1}\left[u_{i,n}(t_1^+) - u_{i,n}(t_1)\right] - e^{\lambda_{i,n}t_2}\left[u_{i,n}(t_2^+) - u_{i,n}(t_2)\right] - \cdots -$$

$$- e^{\lambda_{i,n}t_p}\left[u_{i,n}(t_p^+) - u_{i,n}(t_p)\right] + e^{\lambda_{i,n}t} u_{i,n}(t) = -u_{i,n}(0) - \sum_{0<t_m<t}^{p} e^{\lambda_{i,n}t_m} I_{m,i,n} + e^{\lambda_{i,n}t} u_{i,n}(t).$$

Hence, we obtain that

$$u_{i,n}(t) = u_{i,n}(0)e^{-\lambda_{i,n}t} + \int\limits_0^t e^{-\lambda_{i,n}(t-s)} f_{i,n}(s,\cdot)ds + \sum_{0<t_m<t}^{p} e^{-\lambda_{i,n}(t-t_m)} I_{m,i,n}(\cdot). \tag{22}$$

For given function $\varphi(x)$ in (3) we set $\varphi(x) = \varphi_1(x) + \varphi_2(x)$. Now, supposing that the functions $\varphi_1(x)$ and $\varphi_2(x)$ are expanding into a Fourier series and using the Fourier coefficients (16), from condition (3) we obtain

$$u_{i,n}(0) = \int\limits_{-1}^{1} U_i(0,x)\, \vartheta_{i,n}(x)\, dx = \int\limits_{-1}^{1} \varphi_i(x)\, \vartheta_{i,n}(x)\, dx = \varphi_{i,n}, \quad i = 1,2. \tag{23}$$

Using initial value conditions (23), we found the unknown coefficients $u_{i,n}(0)$ in presentations (22). Then we have

$$u_{i,n}(t) = \varphi_{i,n}e^{-\lambda_{i,n}t} + \int\limits_0^t e^{-\lambda_{i,n}(t-s)} f_{i,n}(s,\cdot)ds + \sum_{0<t_m<t}^{p} e^{-\lambda_{i,n}(t-t_m)} I_{m,i,n}(\cdot). \tag{24}$$

Representations (24) are Fourier coefficients of formal solution to problem (1)–(4). So, taking into account (17) and (18), we rewrite it as

$$u_{i,n}(t) = A(t;u_{i,n}) \equiv \varphi_{i,n}e^{-\lambda_{i,n}t} +$$

$$+ \int\limits_0^t e^{-\lambda_{i,n}(t-s)} \int\limits_{-1}^{1} f_i\left(s,x,\int\limits_{-1}^{1} K(y)\sum_{j=1}^{\infty} u_{i,j}(s)\,\vartheta_{i,j}(y)dy\right)\vartheta_{i,n}(x)\,dxds +$$

$$+ \sum_{0<t_m<t}^{p} e^{-\lambda_{i,n}(t-t_m)} \int\limits_{-1}^{1} I_{m,i}\left(x,\int\limits_{-1}^{1} G(y)\sum_{j=1}^{\infty} u_{i,j}(t_m)\,\vartheta_{i,j}(y)dy\right)\vartheta_{i,n}(x)\,dx. \tag{25}$$

Representation (25) is the countable system of nonlinear functional equations. Substituting representation (25) into Fourier series (15), we obtain a formal solution of problem (1)–(4) on the domain Ω

$$U_i(t,x) = \sum_{n=1}^{\infty}\vartheta_{i,n}(x)\left[\varphi_{i,n}e^{-\lambda_{i,n}t} +\right.$$

$$+ \int\limits_0^t e^{-\lambda_{i,n}(t-s)} \int\limits_{-1}^{1} f_i\left(s,x,\int\limits_{-1}^{1} K(y)\sum_{j=1}^{\infty} u_{i,j}(s)\,\vartheta_{i,j}(y)dy\right)\vartheta_{i,n}(x)\,dxds +$$

$$\left. + \sum_{0<t_m<t}^{p} e^{-\lambda_{i,n}(t-t_m)} \int\limits_{-1}^{1} I_{m,i}\left(x,\int\limits_{-1}^{1} G(y)\sum_{j=1}^{\infty} u_{i,j}(t_m)\,\vartheta_{i,j}(y)dy\right)\vartheta_{i,n}(x)\,dx\right]. \tag{26}$$

3. Solvability of the countable system of nonlinear functional equations

Let us investigate the countable system of nonlinear functional equations (25) in the questions of unique solvability. Consider the following Banach spaces, which we will use in our further actions: the space $B_2[0,T]$ of function sequences $\{u_n(t)\}_{n=1}^{\infty}$ on the segment $[0,T]$ with the norm

$$\| u(t) \|_{B_2[0,T]} = \sqrt{\sum_{n=1}^{\infty}\left(\max_{t\in[0,T]} | u_n(t) |\right)^2} < \infty;$$

the Hilbert coordinate space ℓ_2 of number sequences $\{\varphi_n\}_{n=1}^{\infty}$ with the norm

$$\| \varphi \|_{\ell_2} = \sqrt{\sum_{n=1}^{\infty} | \varphi_n |^2} < \infty;$$

the space $L_2[-1,1]$ of square-integrable functions on an interval $[-1,1]$ with norm

$$\| \vartheta(x) \|_{L_2[-1,1]} = \sqrt{\int\limits_{-1}^{1} | \vartheta(x) |^2 \, dx} < \infty;$$

the space $PC_{B_2}([0,T];\mathbb{R}) = \{u_n : [0,T] \to \mathbb{R};\ \vec{u}(t) \in (B_2[t_m, t_{m+1}];\mathbb{R}),\ m = 0,1,\ldots,p\}$ with the norm $\|\vec{u}(t)\|_{PC_{B_2}[0,T]} = \max\{\|\vec{u}(t)\|_{B_2[t_m,t_{m+1}]},\ m = 0,1,2,\ldots,p\}$, where $u_n(t_m^+)$ and $u_n(t_m^-)$ $(m = 0,1,\ldots,p)$ exist and are bounded; $u_n(t_m^-) = u_n(t_m)$.

Smoothness conditions. Let be $\varphi_i(x) \in C^4[-1,1]$, $f_i(t,x,\cdot)$, $I_{m,i}(x,\cdot) \in C_x^2[-1,1]$. Then, we integrate by parts (23) four times on the variable x, (17) and (18) two times on the variable x and we obtain the results

$$|\varphi_{i,n}| \leqslant \frac{|\varphi_n^{(IV)}|}{\lambda_{i,n}^4}, \quad |f_{i,n}(t,\cdot)| \leqslant \frac{|f''_{n,i}(t,\cdot)|}{\lambda_{i,n}^2}, \quad |I_{m,i,n}(t,\cdot)| \leqslant \frac{|I''_{m,i,n}(\cdot)|}{\lambda_{i,n}^2},$$

where

$$\varphi_{i,n}^{(IV)} = \int\limits_{-1}^{1} \frac{\partial^4 \varphi_i(y)}{\partial y^4} \vartheta_{i,n}(y)\, dy, \quad f''_{i,n}(t,\cdot) = \int\limits_{-1}^{1} \frac{\partial^2 f_i(t,y,\cdot)}{\partial y^2} \vartheta_{i,n}(y)\, dy,$$

$$I''_{m,i,n}(\cdot) = \int\limits_{-1}^{1} \frac{\partial^2 I_{m,i}(y,\cdot)}{\partial y^2} \vartheta_{i,n}(y)\, dy, \ \ i = 1,2.$$

Here it is not difficult to prove that Bessel's inequalities are valid:

$$\left\|\varphi_i^{(IV)}\right\|_{\ell_2} \leqslant \left\| \frac{\partial^4 \varphi_i(x)}{\partial x^4} \right\|_{L_2[-1,1]}, \quad \|f_i''(t,\cdot)\|_{\ell_2} \leqslant \left\| \frac{\partial^2 f_i(t,x,\cdot)}{\partial x^2} \right\|_{L_2[-1,1]}$$

$$\left\|I''_{m,i}(\cdot)\right\|_{\ell_2} \leqslant \left\| \frac{\partial^2 I_{m,i}(x,\cdot)}{\partial x^2} \right\|_{L_2[-1,1]}, \quad i = 1,2.$$

Theorem 1. *Let the smoothness conditions be fulfilled. If the following conditions are fulfilled:*

$$\max_{m=\overline{1,p}} \max_{t\in[t_m,t_{m+1}]} \sqrt{\int\limits_{-1}^{1} \left[I_{i,m}\left(x, \int\limits_{-1}^{1} G(y) \sum_{j=1}^{\infty} u_{i,j}^0(t)\, \vartheta_{i,j}(y) dy \right) \right]^2 dx} = \text{const} < \infty,$$

$$\max \max_{t\in[t_m,t_{m+1}]} \sqrt{\int\limits_{-1}^{1} \left[f_i\left(t, x, \int\limits_{-1}^{1} K(y) \sum_{j=1}^{\infty} u_{i,j}^0(t)\, \vartheta_{i,j}(y) dy \right) \right]^2 dx} = \text{const} < \infty,$$

$$\left| I_{m,i}\left(x, \int\limits_{-1}^{1} G(y) \sum_{j=1}^{\infty} u_{i,j}^{\tau}(t_m)\, \vartheta_{i,j}(y) dy \right) - I_{m,i}\left(x, \int\limits_{-1}^{1} G(y) \sum_{j=1}^{\infty} u_{i,j}^{\tau-1}(t_m) \vartheta_{i,j}(y) dy \right) \right| \leqslant$$

$$\leqslant P_{m,i}(x) \left| \int\limits_{-1}^{1} G(y) \sum_{j=1}^{\infty} \left| u_{i,j}^{\tau}(t_m) - u_{i,j}^{\tau-1}(t_m) \right| \vartheta_{i,j}(y) dy \right|, \ \ 0 < P_{m,i}(x) \in C[-1,1],$$

$$\left| f_i\left(t,x,\int\limits_{-1}^{1}K(y)\sum_{j=1}^{\infty}u_{i,j}^{\tau}(t)\,\vartheta_{i,j}(y)dy\right)-f_i\left(t,x,\int\limits_{-1}^{1}K(y)\sum_{j=1}^{\infty}u_{i,j}^{\tau-1}(t)\,\vartheta_{i,j}(y)dy\right)\right|\leqslant$$

$$\leqslant Q_i(x)\left|\int\limits_{-1}^{1}K(y)\sum_{j=1}^{\infty}\left|\,u_{i,j}^{\tau}(t)-u_{i,j}^{\tau-1}(t)\,\right|\vartheta_{i,j}(y)dy\right|,\ \ 0<Q_i(x)\in C[-1,1],$$

$$\rho_i=C\cdot M_{3,i}\left[M_2\left\|P_i(x)\right\|_{L_2[-1,1]}\left\|G(x)\right\|_{L_2[-1,1]}+\right.$$

$$\left.+M_{1,i}\left\|Q_i(x)\right\|_{L_2[-1,1]}\left\|K(x)\right\|_{L_2[-1,1]}\right]<1, \tag{27}$$

$$\left\|P_i(x)\right\|_{L_2[-1,1]}=\max_{m=1,\dots,p}\left\|P_{m,i}(x)\right\|_{L_2[-1,1]},\quad i=1,2,$$

where $C, M_{1,i}, M_2, M_{3,i}$ *are some constants, then the countable system of nonlinear functional equations* (25) *is uniquely solvable in the space* $PC_{B_2}[0,T]$. *In this case, the desired solution can be founded by the following iteration process:*

$$\begin{cases} u_{i,n}^{0}(t)=\varphi_{i,n}e^{-\lambda_{i,n}t}, \\ u_{i,n}^{\tau+1}(t)=A\left(t;u_{i,n}^{\tau}\right), \quad i=1,2, \quad \tau=0,1,2,\dots \end{cases} \tag{28}$$

Proof. We use the method of contraction maps in combination with the method of successive approximations in the space $PC_{B_2}[0,T]$. By virtue of conditions of the theorem and applying the Cauchy — Schwartz inequality and the Bessel inequality, we obtain from approximations (28) that the following estimate is valid:

$$\left\|\vec{u}_i^{\,0}(t)\right\|_{PC_{B_2}[0,T]}\leqslant\max_{m=\overline{1,p}}\sum_{n=1}^{\infty}\max_{t\in[t_m,t_{m+1}]}|u_{i,n}^{0}(t)|\leqslant$$

$$\leqslant\max_{m=\overline{1,p}}\sum_{n=1}^{\infty}|\varphi_{i,n}|\max_{t\in[t_m,t_{m+1}]}\{e^{-\lambda_{i,n}t}\}\leqslant\sum_{n=1}^{\infty}\frac{|\varphi_n^{(IV)}|}{\lambda_{i,n}^4}\leqslant$$

$$\leqslant\sum_{n=1}^{\infty}\frac{|\varphi_{i,n}^{(IV)}|}{\lambda_{i,n}^4}\leqslant\sqrt{\sum_{n=1}^{\infty}\frac{1}{\lambda_{i,n}^8}}\sqrt{\sum_{n=1}^{\infty}|\varphi_{i,n}^{(IV)}|^2}=\delta_{1,i}<\infty,\quad i=1,2, \tag{29}$$

where $\lambda_{1,n}=(1+\varepsilon)\pi^2(n+0.5)^2$, $\lambda_{2,n}=(1-\varepsilon)\pi^2n^2$, $|\varepsilon|<1$.

In next estimate we take into account that

$$\int\limits_{0}^{t}e^{-\lambda_{i,n}(t-s)}ds\leqslant\frac{1}{\lambda_{i,n}}\left[1-e^{-\lambda_{i,n}t}\right]\leqslant\frac{1}{\lambda_{i,n}}M_{1,i},\quad 0<M_{1,i}=\text{const}.$$

Then, by virtue of estimate (29), applying the Cauchy–Schwartz inequality and the Bessel inequality, for the first difference of approximations (28) we obtain with

$$S_i(t,y)\equiv\sum_{j=1}^{\infty}u_{i,j}^{0}(t)\vartheta_{i,j}(y):$$

$$\left\|\vec{u}_i^{\,1}(t)-\vec{u}_i^{\,0}(t)\right\|_{PC_{B_2}[0,T]}\leqslant\max_{m=\overline{1,p}}\sum_{n=1}^{\infty}\max_{t\in[t_m,t_{m+1}]}|u_{i,n}^{1}(t)-u_{i,n}^{0}(t)|\leqslant$$

$$\leqslant \max_{m=\overline{1,p}} \sum_{n=1}^{\infty} \max_{t\in[t_m,t_{m+1}]} \left| \sum_{0<t_m<t}^{p} e^{-\lambda_{i,n}(t-t_m)} \int_{-1}^{1} I_{m,i}\left(x, \int_{-1}^{1} G(y) S_i(t_m,y)dy\right) \vartheta_{i,n}(x)dx \right| +$$

$$+C \max_{m=\overline{1,p}} \sum_{n=1}^{\infty} \sup_{t\in(t_m,t_{m+1}]} \left| \int_{0}^{t} e^{-\lambda_{i,n}(t-s)} \int_{-1}^{1} f_i\left(s,x, \int_{-1}^{1} K(y) S_i(s,y)dy\right) \vartheta_{i,n}(x)dxds \right| \leqslant$$

$$\leqslant C \max_{m=\overline{1,p}} \sum_{n=1}^{\infty} \frac{1}{\lambda_{i,n}} \sup_{t\in(t_m,t_{m+1}]} \sum_{m=1}^{p} \frac{1}{t-t_m} \left| \int_{-1}^{1} I_{m,i}\left(x, \int_{-1}^{1} G(y) S_i(t,y)dy\right) \vartheta_{i,n}(x)dx \right| +$$

$$+C\cdot M_{1,i} \max_{m=\overline{1,p}} \sum_{n=1}^{\infty} \frac{1}{\lambda_{i,n}} \sup_{t\in(t_m,t_{m+1}]} \left| \int_{-1}^{1} f_i\left(t,x, \int_{-1}^{1} K(y) S_i(t,y)dy\right) \vartheta_{i,n}(x)dx \right| \leqslant$$

$$\leqslant M_2 \sqrt{\sum_{n=1}^{\infty} \frac{C^2}{\lambda_{i,n}^2}} \max_{m=\overline{1,p}} \max_{t\in[t_m,t_{m+1}]} \sqrt{\sum_{n=1}^{\infty} \left[\int_{-1}^{1} I_{m,i}\left(x, \int_{-1}^{1} G(y) S_i(t,y)dy\right) \vartheta_{i,n}(x)dx \right]^2} +$$

$$+M_{1,i} \sqrt{\sum_{n=1}^{\infty} \frac{C^2}{\lambda_{i,n}^2}} \max_{m=\overline{1,p}} \max_{t\in[t_m,t_{m+1}]} \sqrt{\sum_{n=1}^{\infty} \left[\int_{-1}^{1} f_i\left(t,x, \int_{-1}^{1} K(y) S_i(t,y)dy\right) \vartheta_{i,n}(x)dx \right]^2} \leqslant$$

$$\leqslant C\cdot M_2 M_{3,i} \max_{m=\overline{1,p}} \left\| I_{m,i}\left(x, \int_{-1}^{1} G(y) \sum_{j=1}^{\infty} \delta_{1,i}\, \vartheta_{i,j}(y)dy\right) \right\|_{L_2[-1,1]} +$$

$$+C\cdot M_{1,i} M_{3,i} \max_{m=\overline{1,p}} \max_{t\in[t_m,t_{m+1}]} \left\| f_i\left(t,x, \int_{-1}^{1} K(y) \sum_{j=1}^{\infty} \delta_{1,i}\, \vartheta_{i,j}(y)dy\right) \right\|_{L_2[-1,1]} < \infty, \quad (30)$$

where

$$M_2 = \sup_{t\in(t_m,t_{m+1}]} \sum_{m=1}^{p} \frac{1}{t-t_m}, \quad M_{3,i} = \sqrt{\sum_{n=1}^{\infty} \frac{1}{\lambda_{i,n}^2}}, \quad 1<C=\text{const},$$

$$\lambda_{1,n} = (1+\varepsilon)\,\pi^2(n+0.5)^2, \ \ \lambda_{2,n} = (1-\varepsilon)\,\pi^2 n^2, \ \ |\,\varepsilon\,|<1.$$

Continuing this process, similarly to the estimate (30) we obtain

$$\left\| \vec{u}_i^{\,\tau+1}(t) - \vec{u}_i^{\,\tau}(t) \right\|_{PC_{B_2}[0,T]} \leqslant \max_{m=\overline{1,p}} \sum_{n=1}^{\infty} \max_{t\in[t_m,t_{m+1}]} \left| u_{i,n}^{\tau}(t) - u_{i,n}^{\tau-1}(t) \right| \leqslant$$

$$\leqslant C \max_{m=\overline{1,p}} \left| \sum_{n=1}^{\infty} \sum_{0<t_m<t}^{p} \sup_{t\in(t_m,t_{m+1}]} e^{-\lambda_{i,n}(t-t_m)} \int_{-1}^{1} P_{m,i}(x) \times \right.$$

$$\left. \times \left| \int_{-1}^{1} G(y) \sum_{j=1}^{\infty} \left[u_{i,j}^{\tau}(t_m) - u_{i,j}^{\tau-1}(t_m) \right] \vartheta_{i,j}(y)dy \right| \vartheta_{i,n}(x)dx \right| +$$

$$+C \max_{m=\overline{1,p}} \left| \sum_{n=1}^{\infty} \sup_{t\in(t_m,t_{m+1}]} \int_{0}^{t} e^{-\lambda_{i,n}(t-s)} \int_{-1}^{1} Q_i(x) \times \right.$$

$$\times \left| \int_{-1}^{1} K(y) \sum_{j=1}^{\infty} \left[u_{i,j}^{\tau}(s) - u_{i,j}^{\tau-1}(s) \right] \vartheta_{i,j}(y) dy \right| \vartheta_{i,n}(x)\, dxds \Bigg| \leqslant$$

$$\leqslant M_2 \max_{m=\overline{1,p}} \sum_{n=1}^{\infty} \frac{C}{\lambda_{i,n}} \int_{-1}^{1} P_{m,i}(x) \vartheta_{i,n}(x) dx \times$$

$$\times \sum_{j=1}^{\infty} \max_{t\in[t_m,t_{m+1}]} \left| u_{i,j}^{\tau}(t) - u_{i,j}^{\tau-1}(t) \right| \left| \int_{-1}^{1} G(y) \vartheta_{i,j}(y) dy \right| +$$

$$+ M_{1,i} \max_{m=\overline{1,p}} \sum_{n=1}^{\infty} \frac{C}{\lambda_{i,n}} \int_{-1}^{1} Q_i(x) \vartheta_{i,n}(x) dx \times$$

$$\times \sum_{j=1}^{\infty} \max_{t\in[t_m,t_{m+1}]} \left| u_{i,j}^{\tau}(t) - u_{i,j}^{\tau-1}(t) \right| \left| \int_{-1}^{1} K(y) \vartheta_{i,j}(y) dy \right| \leqslant$$

$$\leqslant C \cdot M_2 M_{3,i} \left\| P_i(x) \right\|_{L_2[-1,1]} \left\| \vec{u}_i^{\tau}(t) - \vec{u}_i^{\tau-1}(t) \right\|_{PC_{B_2}[0,T]} \| G(x) \|_{L_2[-1,1]} +$$

$$+ C \cdot M_{1,i} M_{3,i} \left\| Q_i(x) \right\|_{L_2[-1,1]} \left\| \vec{u}_i^{\tau}(t) - \vec{u}_i^{\tau-1}(t) \right\|_{PC_{B_2}[0,T]} \| K(x) \|_{L_2[-1,1]} \leqslant$$

$$\leqslant \rho_i \left\| u_i^{\tau}(t) - u_i^{\tau-1}(t) \right\|_{PC_{B_2}[0,T]}, \tag{31}$$

$$\rho_i = C \cdot M_{3,i} \left[M_2 \| P_i(x) \|_{L_2[-1,1]} \| G(x) \|_{L_2[-1,1]} + M_{1,i} \| Q_i(x) \|_{L_2[-1,1]} \| K(x) \|_{L_2[-1,1]} \right],$$

$$\| P_i(x) \|_{L_2[-1,1]} = \max_{m=1,\dots,p} \| P_{m,i}(x) \|_{L_2[-1,1]}, \quad i = 1, 2.$$

According to (27), $\rho_i < 1$. Consequently, it follows from estimate (31) that the operator on the right-hand side of the countable system of nonlinear functional equations (25) is contracting. It follows from estimates (29)–(31) that there is a unique fixed point, which is a solution to the countable system of functional equations (25) in space $PC_{B_2}[0,T]$. Theorem 1 is proved. □

4. Uniform convergence of Fourier series

Theorem 3. *Let the conditions of Theorem 1 be fulfilled. Then the unknown function $U(t,x) = U_1(t,x) + U_2(t,x)$ of mixed impulsive problem (1)–(4) is defined by Fourier series (26). Series (26), its two derivatives on the variable x and the first derivative on the variable t converge on the domain Ω.*

Proof. Let $\vec{u}_i(t) \in PC_{B_2}[0,T]$ be unique solution of countable system (25). As in the case of estimates (29) and (30), we obtain

$$|U_i(t,x)| \leqslant \sum_{n=1}^{\infty} |\vartheta_{i,n}(x)| \left(\max_{m=\overline{1,p}} \sum_{n=1}^{\infty} |\varphi_{i,n}| \max_{t\in[t_m,t_{m+1}]} \left\{ e^{-\lambda_{i,n} t} \right\} + \right.$$

$$\left. + C \cdot \max_{m=\overline{1,p}} \sum_{n=1}^{\infty} \sup_{t\in(t_m,t_{m+1})} \left| \sum_{0<t_m<t}^{p} e^{-\lambda_{i,n}(t-t_m)} \int_{-1}^{1} I_{m,i}(x,\cdot)\, \vartheta_{i,n}(x) dx \right| + \right.$$

$$+C\cdot\max_{m=\overline{1,p}}\sum_{n=1}^{\infty}\sup_{t\in(t_m,t_{m+1}]}\left|\int_0^t e^{-\lambda_{i,n}(t-s)}\int_{-1}^{1} f_i\left(s,x,\cdot\right)\vartheta_{i,n}(x)dxds\right|\Bigg)\leqslant$$

$$\leqslant\sqrt{\sum_{n=1}^{\infty}\frac{1}{\lambda_{i,n}^8}}\|\varphi_i^{(IV)}\|_{\ell_2}+C\cdot M_2M_{3,i}\max_{m=\overline{1,p}}\|\,I_{m,i}\left(x,\cdot\right)\|_{L_2[-1,1]}+$$

$$+C\cdot M_{1,i}M_{3,i}\max_{m=\overline{1,p}}\max_{t\in[t_m,t_{m+1}]}\|\,f_i\left(t,x,\cdot\right)\|_{L_2[-1,1]}<\infty. \tag{32}$$

Estimate (32) implies the absolute and uniform convergence of series (26). Analogously the smoothness of solution (26) may be proved by the aid of the smoothness conditions. Theorem 3 is proved. □

5. Conclusion

In the domain $\Omega=\{t\in(0,T),\ t\neq t_m,\ 0<t_m<T,\ m=1,2,\ldots,p,\ x\in(-1,1)\}$ we consider a parabolic type nonlinear differential equation (1) with nonlinear impulsive effects and involution. Dirichlet boundary value conditions (2) with respect to spatial variable x are used. First, homogeneous problem (5), (6) is considered. This problem is replaced with spectral problem (7), (8). Eigenvalues (10), (13) and eigenfunctions (11), (14) of spectral problem (7), (8) are found. The Fourier series method of variables separation is applied. The countable system of nonlinear functional equations (25) is obtained. Theorem 1 on a unique solvability of countable system of nonlinear functional equations (25) is proved. The method of successive approximations is used in combination with the method of contraction mappings. The solution of mixed problem (1)–(4) is obtained in the form of Fourier series (26). The absolute and uniform convergence of Fourier series (26) are proved (Theorem 3).

We will need this work in our future work to investigate other type partial differential equations of the arbitrary order with impulse effects.

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СМЕШАННАЯ ЗАДАЧА ДЛЯ НЕЛИНЕЙНОГО ИМПУЛЬСНОГО ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ ПАРАБОЛИЧЕСКОГО ТИПА

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Рассматривается нелинейное импульсное уравнение в частных производных параболического типа с нелинейными импульсными условиями. По пространственной переменной использованы граничные условия типа Дирихле, найдены собственные значения и собственные функции спектральной задачи. Применён метод Фурье разделения переменных. Получена счётная система нелинейных функциональных уравнений относительно коэффициентов Фурье неизвестной функции. Доказана теорема об однозначной разрешимости счётной системы нелинейных функциональных уравнений. При этом был использован метод последовательных приближений. Получены критерии единственности и существования решения нелинейной импульсной смешанной задачи. Решение смешанной задачи получено в виде ряда Фурье. Доказана абсолютная и равномерная сходимость рядов Фурье.

Ключевые слова: *смешанная задача, нелинейное параболическое уравнение, нелинейные импульсные условия, инволюция, однозначная разрешимость.*

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