Chelyabinsk Physical and Mathematical Journal. 2020. Vol. 5, iss. 2. P. 194–201.

DOI: 10.24411/2500-0101-2020-15206

DISCRETE MAGNETIC BREATHERS IN MONOAXIAL CHIRAL HELIMAGNET

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Intrinsic localized spin modes, or discrete breathers, are investigated in the forced ferromagnetic state of a monoaxial chiral helimagnet. The approximate solution of these excitations are obtained with the aid of discrete equations of spin dynamics. Conditions on the frequency of the breathers and on the easy-plane anisotropy are established under which the breathers are possible. In the presence of Dzyaloshinskii — Moryia interaction the localized spin modes become spatially modulated and, as a consequence, acquire the chirality. Energy of these excitations, including a pinning potential, is calculated.

Keywords: *discrete breather, chiral helimagnet.*

Introduction

According to generally accepted definition, a breather is a localized in space and periodic in time solution of either continous media equations or discrete lattice equations. The breather solutions were found, for example, for the exactly solvable sine-Gordon equation [1–3]. The discrete nonlinear lattices also reveal spatially localized oscillating modes. It was found out that the nonlinearity and the discretness are two pivotal ingredients supporting these excitations, which were named as discrete breathers or intrinsic localized modes [4; 5]. Most of the previous studies were focused on the discrete breathers in lattice models or crystals that constitutes nonlinear discrete systems [6–8]. However, there remains a need to find these excitations in magnetic models. Several studies were undertaken to verify discrete breathers in ferromagnets and antiferromagnets with an easy-plane anisotropy [9–11]. It has been confirmed that these intrinsic localized modes have high frequencies, above the maximum frequency of the spin-wave spectrum.

The purpose of this paper is to discuss localized spin modes in the model of monoaxial chiral helimagnet. Folllowing the idea suggested in [9] we address the so-called phase of forced ferromagnetism, where in the ground state all spins align with an external

The work is supported by the RFBR, grant no. 20-02-00213; by the Government of the Russian Federation, Act 211 of 16.03.2013, contracts no. 02.A03.21.0011 and no. 02.A03.21.0006; by the Foundation for the Advancement of Theoretical Physics and Mathematics "Basis", grant no. 17-11-107.

magnetic field applied along the chiral axis [12]. Our analysis shows that the intrinsic localized modes exist and the Dzyaloshinskii — Moryia (DM) interaction does not hinder their emergence.

1. The model

We explore the spin Hamiltonian

$$\mathcal{H} = -2J\sum_n \mathbf{S}_n \cdot \mathbf{S}_{n+1} + A\sum_n (S_n^z)^2 - H_0 \sum_n S_n^z + \mathcal{D}\sum_n [\mathbf{S}_n \times \mathbf{S}_{n+1}]_z \,,$$

where the first term corresponds to the exchange coupling of the strength $J > 0$ along the chiral axis (the z-axis), the $\mathbf{S}_n$ is the spin vector of the nth site. The second term means the easy-plane anisotropy with the constant $A > 0$, so the xy-plane being the easy-plane of magnetization. The third term describes the Zeeman coupling with the external magnetic field H_0 directed along the z-axis. The last term stands for Dzyaloshinskii — Moryia interaction along the chiral axis with the strength D. It is supposed that the field H_0 exceeds the treshold value $H_{cr} = 2S\left(\sqrt{4J^2 + D^2} - J + A\right)$, thus the phase of forced ferromagnetism is stabilized [12].

The equation of motion for the operator $S_n^+ = S_n^x + iS_n^y$

$$i\hbar \frac{dS_n^+}{dt} = \left[S_n^+, H\right]$$

has the explicit form

$$i\hbar \frac{dS_n^+}{dt} = H_0 S_n^+ - A\left(S_n^+ S_n^z + S_n^z S_n^+\right) +$$

$$+2J\left[S_n^+\left(S_{n+1}^z + S_{n-1}^z\right) - S_n^z\left(S_{n-1}^+ + S_{n+1}^+\right)\right] + i\mathcal{D}S_n^z\left(S_{n-1}^+ - S_{n+1}^+\right).$$

By introducing the normalized classical variables, $s_n^\pm = S_n^\pm/S$ and $s_n^z = \sqrt{1 - s_n^+ s_n^-}$, we obtain

$$\frac{i\hbar}{2JS}\frac{d}{dt}s_n^+ = \frac{H_0}{2JS}s_n^+ - 2Bs_n^+ s_n^z + s_n^+\left(s_{n+1}^z + s_{n-1}^z\right) -$$

$$-s_n^z\left(s_{n-1}^+ + s_{n+1}^+\right) + i\frac{D}{2J}s_n^z\left(s_{n-1}^+ - s_{n+1}^+\right), \quad (1)$$

where $B = A/2J$.

We look for solutions in the form $s_n^+(t) = s_n(t)\exp\left(-i\Omega t + ikna\right)$ and $s_n^z = \sqrt{1 - s_n^2}$, where a is a lattice constant, k is a wave number that will be specified later, Ω is the frequency. Notice, we admit that s_n depends on time in contrast to the analysis of discrete breathers in ferromagnets [9].

The real and imaginary parts of Eq. (1) give, respectively,

$$\Omega s_n = s_n\left(\sqrt{1 - s_{n+1}^2} + \sqrt{1 - s_{n-1}^2}\right) - \sqrt{1 - s_n^2}\left(s_{n-1} + s_{n+1}\right)\cos ka -$$

$$-2Bs_n\sqrt{1 - s_n^2} + \frac{D}{2J}\sqrt{1 - s_n^2}\left(s_{n-1} + s_{n+1}\right)\sin ka \quad (2)$$

and

$$\frac{\hbar}{2JS}\frac{ds_n}{dt} = \sqrt{1 - s_n^2}\left(s_{n-1} - s_{n+1}\right)\sin ka + \frac{D}{2J}\sqrt{1 - s_n^2}\left(s_{n-1} - s_{n+1}\right)\cos ka. \quad (3)$$

By introducing the phase δ conditioned by

$$\cos\delta = \frac{1}{\sqrt{1+\frac{D^2}{4J^2}}}, \quad \sin\delta = \frac{\frac{D}{2J}}{\sqrt{1+\frac{D^2}{4J^2}}},$$

we recast the system (2), (3) as

$$\Omega s_n = -2Bs_n\sqrt{1-s_n^2} + s_n\left(\sqrt{1-s_{n+1}^2}+\sqrt{1-s_{n-1}^2}\right) -$$

$$-\sqrt{1-s_n^2}\,(s_{n-1}+s_{n+1})\sqrt{1+\frac{D^2}{4J^2}}\cos(ka+\delta), \tag{4}$$

$$\frac{ds_n}{d\tau} = \sqrt{1-s_n^2}\,(s_{n-1}-s_{n+1})\sqrt{1+\frac{D^2}{4J^2}}\sin(ka+\delta). \tag{5}$$

In the last line the dimensionless time is determined, $\tau = t/t_0$, where $t_0 = \hbar/(2JS)$.

2. Discrete breathers

To find breather solutions we assume $ds_n/d\tau = 0$. Firstly, we note that $\delta = 0$ at $D = 0$. Then, as it follows from Eq. (5), $\sin(ka) = 0$ or $k = 0$. Eq. (4) is reduced to

$$\Omega s_n = -2Bs_n\sqrt{1-s_n^2} + s_n\left(\sqrt{1-s_{n+1}^2}+\sqrt{1-s_{n-1}^2}\right) - \sqrt{1-s_n^2}\,(s_{n-1}+s_{n+1}) \tag{6}$$

that has been investigated in the work [9].

For non-zero D a solution exists provided $\sin(ka+\delta) = 0$, i.e.

$$k = -\frac{1}{a}\tan^{-1}\left(\frac{D}{2J}\right). \tag{7}$$

This relationship relates the wavenumber of the discrete breather with the pitch of spiral order in the conical phase, which is stable at $H < H_{\rm cr}$.

As it has been demonstrated in the previous studies of ferromagnets, the discrete breathers are featured by an alternating sign of the s_n arrangement, i.e. by staggered magnetization. To introduce a continuum description of the breather excitations, it is convenient to define the smoothly varying envelope function $\psi(z) = (-1)^n s_n$, where $z = na$.

Taking into account the expressions

$$s_n = (-1)^n\psi(z), \quad s_{n\pm1} = (-1)^{n\pm1}\left[\psi(z) \pm a\frac{d\psi}{dz} + \frac{a^2}{2}\frac{d^2\psi}{dz^2}\right],$$

$$\sqrt{1-s_n^2} \approx 1 - \frac{1}{2}\psi^2(z), \quad \sqrt{1-s_{n\pm1}^2} \approx 1 - \frac{1}{2}\psi^2(z) \mp a\psi(z)\frac{d\psi(z)}{dz},$$

and substituting them into Eq. (4) we obtain eventually the classical Duffing equation

$$\frac{d^2\psi}{d\tilde{z}^2} - \alpha\psi + \beta\psi^3 = 0,$$

if to ignore nonlinear terms involving $d\psi/dz$ and $d^2\psi/dz^2$. Here, the dimensionless coordinate $\tilde{z} = z/a$ is introduced and the notions

$$\alpha = \frac{2B + \Omega - 2 - 2\sqrt{1+\frac{D^2}{4J^2}}\cos(ka+\delta)}{\sqrt{1+\frac{D^2}{4J^2}}\cos(ka+\delta)}, \tag{8}$$

$$\beta = \frac{B - 1 - \sqrt{1 + \frac{D^2}{4J^2}}\cos(ka+\delta)}{\sqrt{1 + \frac{D^2}{4J^2}}\cos(ka+\delta)} \tag{9}$$

are adopted. We note that the results of [9], $\alpha = 2B + \Omega - 4$ and $\beta = B - 2$, are reproduced from Eqs. (8), (9), if to take $D = 0$ and $ka + \delta = 0$.

Given the imposed boundary conditions $\psi(\pm\infty) = 0$ and $(d\psi/dz)|_{z=\pm\infty} = 0$ valid for localized breather solutions we obtain

$$\int \frac{d\psi}{\sqrt{\alpha\psi^2 - \frac{1}{2}\beta\psi^4}} = \pm\left(\tilde{z} - \tilde{z}_0\right),$$

where $\tilde{z}_0$ is a constant of integration. Making use the change $y^2 = \alpha - \frac{1}{2}\beta\psi^2$, we obtain

$$\int \frac{dy}{y^2 - \alpha} = \pm\left(\tilde{z} - \tilde{z}_0\right).$$

The condition $\alpha > 0$ must be fulfilled to get a localized solution. Eventually, we find

$$\psi(z) = \frac{\psi_m}{\cosh\left[\frac{\psi_m}{2a}\sqrt{2\beta}\,(z - z_0)\right]}, \tag{10}$$

where $\psi_m = \sqrt{2\alpha/\beta}$, and, apparently, there arises $\beta > 0$.

In the presence of the Dzyaloshinskii — Moryia interaction the solution exists provided $\sin(ka+\delta) = 0$, or $\cos(ka+\delta) = \pm 1$. The upper sign yields conditions for the frequency

$$\Omega > 2 + 2\sqrt{1 + \frac{D^2}{4J^2}} - 2B,$$

and the easy-plane anisotropy

$$B > 1 + \sqrt{1 + \frac{D^2}{4J^2}}.$$

We note that the opposite sign brings about

$$\Omega < 2 - 2\sqrt{1 + \frac{D^2}{4J^2}} - 2B, \quad B < 1 - \sqrt{1 + \frac{D^2}{4J^2}},$$

but the latter is inconsistent to the requirement of the easy-plane anisotropy, $B > 0$, thus, fixing $\cos(ka+\delta) = 1$.

Notice that the breather solution (10) accumulates not only magnon density but a topological charge in contrast to the case of usual ferromagnet. Its value depends on the balance between the scale of breather localization, $l = 4a/(\psi_m\sqrt{2\beta}) = 2a/\sqrt{\alpha}$, and the pitch of the spiral, k, given by Eq. (7). By definition, the topological charge protected by the DM interaction is

$$\mathcal{Q} = \frac{\Delta\varphi}{2\pi} = \frac{kl}{2\pi} = -\frac{1}{\pi\sqrt{\alpha}}\tan^{-1}\left(\frac{D}{2J}\right), \tag{11}$$

where $\Delta\varphi$ is the total spin rotation angle along the path l. The spin arrangement of the discrete breather is presented in Fig. 1.

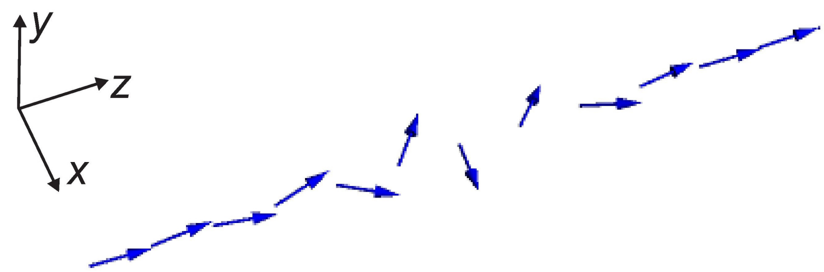

Fig. 1. Snapshot of discrete breather of the chiral helimagnet in the forced ferromagnetic state

3. Energy of the discrete breather

By using the envelope function $s_n = (-1)^n \psi(z)$ the energy stored in the breather amounts to

$$E - E_0 = \sqrt{4J^2 + D^2} S^2 \sum_n \psi(na)\psi(na+a) + \left(2JS^2 + \frac{1}{2} H_0 S - AS^2 \right) \sum_n \psi^2(na), \quad (12)$$

where $E_0 = -2JS^2 N - H_0 SN + AS^2 N$ is the energy of the ferromagnetic background.

The direct calculation of the sum in the continuum limit gives

$$\sum_n \psi^2(na) = \sum_n \psi^2(na + a) = \int_{-\infty}^{\infty} \frac{dz}{a} \psi^2(z) = \frac{4\sqrt{\alpha}}{\beta},$$

$$\sum_n \psi(na)\psi(na + a) \approx \int_{-\infty}^{\infty} \frac{dz}{a} \psi(z) \left[\psi(z) + a \frac{d}{dz} \psi(z) \right] = \int_{-\infty}^{\infty} \frac{dz}{a} \psi^2(z) = \frac{4\sqrt{\alpha}}{\beta}.$$

Therefore, the energy of the chiral breather is

$$\Delta E^{(1)} = E - E_0 = \frac{4\sqrt{\alpha}}{\beta} \left[\sqrt{4J^2 + D^2} S^2 + 2JS^2 + \frac{1}{2} H_0 S - AS^2 \right]. \quad (13)$$

This expression does not take into account discreteness of the lattice giving rise to a pinning energy [13]. To calculate this contribution we use the continuum solution as

$$f(z - z_0) = \frac{\psi_m}{\cosh \left[\frac{\psi_m}{2a} \sqrt{2\beta} \, (z - z_0) \right]},$$

where dependence on the central coordinate z_0 is clearly indicated, and rewrite (12)

$$E - E_0 = \int dz \left[\sum_n \delta\left(z - na\right) \right] \times$$

$$\times \left[\sqrt{4J^2 + D^2} S^2 f(z - z_0) f(z - z_0 + a) + \left(2JS^2 + \frac{1}{2} H_0 S - AS^2 \right) f^2(z) \right]. \quad (14)$$

By using the identity

$$\sum_n \delta\left(z - na\right) = \frac{1}{a} + \frac{2}{a} \sum_{n=1}^{\infty} \cos\left(\frac{2\pi n z}{a} \right) \quad (15)$$

and substituting it into (14), one can see that the first term of Eq. (15) leads to $\Delta E^{(1)}$ while the second is related with the pinning energy

$$\Delta E^{(2)} = 2\sqrt{4J^2 + D^2} S^2 \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \frac{dz}{a} \cos\left(\frac{2\pi n}{a} \left[z + z_0 \right] \right) f(z) f(z + a) +$$

$$+2\left(2JS^2+\frac{1}{2}H_0S-AS^2\right)\sum_{n=1}^{\infty}\int_{-\infty}^{\infty}\frac{dz}{a}\cos\left(\frac{2\pi n}{a}\left[z+z_0\right]\right)f^2(z).$$

Calculation of the integrals

$$I_{1n}=\int_{-\infty}^{\infty}\frac{dz}{a}\cos\left(\frac{2\pi n}{a}\left[z+z_0\right]\right)f(z)f(z+a)\approx$$

$$\approx\frac{4\pi^2 n}{\beta\sinh\left(\frac{\pi^2 n}{\sqrt{\alpha}}\right)}\left[\cos\left(\frac{2\pi n z_0}{a}\right)+n\sin\left(\frac{2\pi n z_0}{a}\right)\right],$$

$$I_{2n}=\int_{-\infty}^{\infty}\frac{dz}{a}\cos\left(\frac{2\pi n}{a}\left[z+z_0\right]\right)f^2(z)=\frac{4\pi^2 n}{\beta}\frac{\cos\left(\frac{2\pi n z_0}{a}\right)}{\sinh\left(\frac{\pi^2 n}{\sqrt{\alpha}}\right)}$$

yields the result

$$\Delta E^{(2)}=2\sum_{n=1}^{\infty}\left[\sqrt{4J^2+D^2}S^2I_{1n}+\left(2JS^2+\frac{1}{2}H_0S-AS^2\right)I_{2n}\right],$$

where the term with $n=1$ dominates.

4. Conclusions

In summary, we demonstrate in our study that there are intrinsic nonlinear spin excitations, discrete breathers, in the state of forced ferromagnetism of the monoaxial chiral helimagnet.

The authors wish to acknowledge Professor V.D. Buchelnikov for support and attention to the work of the Ekaterinburg and Ufa schools of theoretical physicists in the field of statics and dynamics of magnetic inhomogeneities.

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Article received 06.05.2020
Corrections received 20.05.2020

Челябинский физико-математический журнал. 2020. Т. 5, вып. 2. С. 194–201.

УДК 530.182.1 DOI: 10.24411/2500-0101-2020-15206

ДИСКРЕТНЫЕ МАГНИТНЫЕ БРИЗЕРЫ В ОДНООСНЫХ ХИРАЛЬНЫХ ГЕЛИМАГНЕТИКАХ

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В форсированном ферромагнитном состоянии моноаксиального хирального гелимагнетика исследуются собственные локализованные спиновые моды, или дискретные бризеры. Получено приближённое решение этих возбуждений с помощью дискретных уравнений спиновой динамики. Устанавливаются условия для частоты бризеров и для анизотропии типа легкая плоскость, при которых бризеры возможны. При наличии взаимодействия Дзялошинского — Мории локализованные спиновые моды становятся пространственно модулированными и, как следствие, приобретают киральность. Вычислена энергия этих возбуждений, включая потенциал пиннинга.

Ключевые слова: *дискретный бризер, хиральный гелимагнетик.*

Поступила в редакцию 06.05.2020
После переработки 20.05.2020

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Работа поддержана грантом РФФИ, проект № 20-02-00213; Правительством РФ, Постановление № 211, контракты № 02.A03.21.0011 и № 02.A03.21.0006; Фондом развития теоретической физики и математики «Базис», грант № 17-11-107.